# IM 

## Dutch Circle of Slide Rule Collectors



9th International Meeting of Slide Rule Collectors


19-20 September 2003 Amsterdam, The Netherlands

## PROCEEDINGS IM 2003



# $9{ }^{\text {th }}$ International Meeting of 

## Slide Rule Collectors

## Amsterdam/Breukelen The Netherlands



## Proceedings

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# Introduction by the Chairman of the Dutch Circle of Slide Rule Collectors (NKVR) 

## Chris Hakkaart

## Dear Participants and Partners,

In front of you is the information about the $9^{\text {th }}$ International Meeting of Slide Rule Collectors, which is organised for the third time in The Netherlands. For the Participants there are interesting subjects and presentations as well as the meet-again possibility with their international friends. We welcome the new participants and hope to meet them again in future. And of course the presence of many of our partners is part of the success.

The Organising Committee has prepared - with numerous others - an interesting program. Due to the very special subjects of the speakers, we are convinced that it will become again a valuable International Meeting. A large part of this IM will be dedicated to the still available knowledge and experience of the recently closed UTO manufacturing company of Flemming Holme in Denmark. The design and the manufacturing process will be explained in detail by the former adviser and now new owner John Kvint. The presentations are accompanied by a special UTO exhibition, made by the expert on this field IJzebrand Schuitema. We have a special word of thanks to Flemming Holme, who offers every participant a number of new UTO Slide Rules.

But also the wide variety and high level of the presentations of the other authors gains attention. Although most of us are primarily interested in Slide Rules, many have also an interest in other calculating devices. There will be presentations about Slide Rules and some very special devices.
The variations in the English language, as used in this Proceeding, mirrors the interest in Slide Rules and related subjects. I am sure we will learn again a lot from each other.

The Organising Committee has also introduced an integrated session for Participants and Partners. The Slide Rule started to develop during the beginning of the seventeenth century. During this period parts of the beautiful city of Amsterdam were built. In this integrated session at Friday afternoon, an explanation will be given about the links between Slide Rules, Proportions, Architecture, seventeenth century, etc. A specialist on Restoration will guide on Saturday a walking tour along and visit of historical buildings and canals in Amsterdam. Explanations will be given about the technical and romantic aspects of these beautiful houses and gardens.

Next year, the first decade of the International Meetings will be reached, which is a real performance when realising that these events are organised and visited by individuals and not by professional organisations, although the quality can certainly compete with other congresses. It will be a challenge for all of us to promote our Subject and to interest younger people, although they need explanation about what a Slide Rule is. But I think it is important to spend energy in it, because governments (such as the Dutch) talk about the future as a knowledge society. However, they actually do not sufficiently promote it, with the consequence that the number of kids, who study technology, is dropping down quickly. I always argue that the use of Slide Rules challenges people more than the number crunching on computers. To make sure, I am not against computers. Maybe we have to introduce the Slide Rule more, to promote a way of thinking required for technical people. We have to see this also in the context of promoting younger people to become a member of our National Organisations. This is important to keep the tradition of International Meetings in future alive. Many of the new participants become regular visitors.

I wish you on behalf of the Organising Committee a pleasant stay in The Netherlands and hope You and your Partner will have two interesting days, with a refreshment of knowledge about calculating instruments and old buildings and an enjoyable time with your friends.

## Acknowledgements

This International Meeting 2003 is only possible with the support and contributions of many individuals and companies. As you can see numerous persons were involved during the preparations. The Organising Committee expresses, also in name of the participants and partners, its thanks for their support.

## Organising Committee

The Organising Committee has, with pleasure and satisfaction, contributed in time, financial support and sponsoring in kind to make this event successful.

## Chris J.A. Hakkaart

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## Support by Individuals

There are many other persons, who have contributed or sponsored in a direct or indirect way to this International Meeting.

First of all the speakers and authors of the papers in the Proceedings, who have contributed by presenting new discoveries in the field of our interest. They have invested time and sometimes even provided financial support for licences from Musea for using information, as Bruce Williams.
Thomas van der Zijden provided general assistance during the first period.
And many others who have helped the Organising Committee on occasional basis, to make the IM 2003 a success. Some of them we have to mention:
The design of the cover which was made in cooperation with Claudia Peters, a cousin of Gerard van Gelswijck.
Our sister organisations abroad, who assisted in supplying information and ideas based on the recent International Meetings.
Henny C. Brouwer, a restoration expert from Delft University of Technology, who explained for participants and partners the situation in the Netherlands during the initial period of the slide rule in the seventeenth century and who guided the tour of the partners to historical Amsterdam.
Wim Granneman, who explained the design of a special Slide Rule for a typical Dutch product.

## List of Company Sponsors for IM2003

This International Meeting could only be organised with the help of many sponsors. They provided financial support or sponsored in kind. The Organising Committee expresses, also in name of the participants, its thanks for their support. The Company sponsors are named in alphabetical order:

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UTO
Danish Slide Rule Manufacturing Company
owned by F. Holme and recently closed

## Summaries and Personal Information of the Authors

## Otto van Poelje

Otto van Poelje studied applied mathematics at the Technical University of Delft. For more than 30 years he has been an employee of Philips, AT\&T and Lucent Technologies. Recently he retired from his work in telecommunications at Lucent.

## Gunter Rules in Navigation



This article addresses the well known Gunter Rule, with its historical background, its scales, and its application in navigation at sea. Besides some other rules for navigation are described.

## Linialen van Gunter bij de Navigatie

Dit artikel bespreekt de bekende liniaal van Gunter. Aan de orde komen de historische achtergronden, de schalen en de toepassingen bij de navigatie op zee. Daarnaast worden enige andere linialen, die bij de navigatie worden gebruikt, beschreven.

## The Slide Rule Catalogue, its Past and Future

This article, written by Otto van Poelje, tells the story of the ever expanding Slide Rule Catalogue, compiled by Herman van Herwijnen. Without this catalogue collecting of slide rules would be a rather unordered process. The catalogue makes it possible for instance to collect slide rules of special scales or special producers and the collector can always see which items he does not have but which he certainly should become the owner of.
Herman began his catalogue in the early nineties, on paper with cheap and rather uncomplicated use of technology. Now the catalogue with digital photo pictures goes over the whole world written by a modern and fast computer on a CD-ROM.

## The Rekenlinialencatalogus, Verleden en Toekomst

Dit artikel, geschreven door Otto van Poelje, vertelt het verhaal van de steeds maar uitgebreider wordende catalogus van rekenlinialen, die wordt samengesteld door Herman van Herwijnen. Zonder deze catalogus zou het verzamelen van rekenlinialen een tamelijk ongeordend proces zijn. De catalogus maakt het bijvoorbeeld mogelijk om gericht rekenlinialen met speciale schalen te verzamelen of linialen van een bepaalde producent en de verzamelaar kan altijd opzoeken welke linialen hij nog niet in zijn verzameling heeft, maar die hij beslist moet zien te bemachtigen. Herman begon zijn catalogus in de vroege jaren negentig, op papier en door middel van toepassing van eenvoudige technologie. Tegenwoordig gaat de catalogus de hele wereld over, met digitale foto's en samengesteld door een snelle computer.

## John Kvint

John Kvint, living in Roskilde, Denmark, Born 1934, high school 1950, mechanic 1954, diploma mechanical engineer 1957, 2-year compulsary service: electronic Lieutenant 1959. Switzerland 1960-1962: Mountain cableways, 1962-1967: Esso Plastic (DK): Plastic machinery 1967-2001: Linex (DK): Product development and production equipment. 1978-1993: development of Linex CAD/CAM and electronic drafting equipment. From 1993 as part-time employment. 1967- 2002: Cretex (DK): slide-chart-designer. 1968-1993: consultant for Inventors office. 1993-: Ownermanager John Kvint Plastteknik. Machining plastics for displays, making equipment for disabled, Linex splines and long rulers, making slide-rules in cooperation w. UTO


## History of UTO and other Danish Slide Rule Firms

This article describes the history of Danish slide rule making, from the end of the $19^{\text {th }}$ century till the seventies of the $20^{\text {th }}$ century, by Linex, DIWA, IWACO, CRETEX and of course UTO, the firm that is so important for the International Meeting 2003..

## De Geschiedenis van UTO en andere Deense Producenten van Rekenlinialen

Dit artikel beschrijft de geschiedenis van de Deense productie van rekenlinialen, vanaf het einde van de $19^{\text {de }}$ eeuw tot de jaren zeventig van de $20^{\text {ste }}$ eeuw. Besproken worden de firma's Linex, DIWA, IWACO, CRETEX en natuurlijk UTO, de producent van rekenlinialen die zo'n belangrijke plaats inneemt op de International Meeting van 2003.

## Design and Printing of Scales

Many collectors of slide rules and historians of technical and scientific instruments are probably not aware of the technical knowledge that is needed for the production of slide rules. The design and printing of scales in particular was a difficult process. The article gives a detailed description of the various production steps that had to be carried out before a scale could be used on a slide rule.

## Het Ontwerpen en Drukken van Schalen voor Rekenlinialen

Veel verzamelaars van rekenlinialen en geschiedkundigen die de historie van technische en wetenschappelijke instrumenten bestuderen zijn waarschijnlijk niet op de hoogte van de technische kennis die benodigd is bij de productie van rekenlinialen. Het ontwerpen en drukken van de rekenliniaalschalen vergde bijzonder veel deskundigheid. Het artikel beschrijft in detail de productiestappen die moesten worden uitgevoerd om een schaal op een rekenliniaal te krijgen.

## UTO Manufacturing Process

After reading this paper it can be understood why slide rules were expensive products for students, engineers and scientists. The article gives a detailed description of the difficult and time consuming production steps in producing a slide rule.

## Het Productieproces van UTO

Na lezing van dit stuk kan men des te beter begrijpen waarom rekenlinialen dure producten waren voor studenten, technici en wetenschappers. Het artikel geeft een gedetailleerde beschrijving van het moeizame en tijd vretende productieproces dat uiteindelijk leidde tot een kant en klare rekenliniaal.

## Bruce O.B. Williams

Bruce O B Williams studied Natural Sciences at Cambridge University, England, and was awarded a MBA Degree from the Harvard Business School in 1959. He has been a Management Consultant in London since 1968,. He has published articles on CheckSums and on Poly-slide Rules. He is now preparing a PhD dissertation on the subject of "Commercial Multiplication" under the supervision of Professor Roger Johnson at Birkbeck College, University of London.


## Ready Reckoners and Tabular Calculators

Before the advent of cheap and easy-to-use electronic calculating devices, commercial calculations were too demanding for slide rules, mechanical machines or logarithm tables, particularly when non-decimal calculations had to be performed. An important answer to these complications was met by Ready Reckoners and Tabular Calculators, which used all kinds of pre-calculated results. Bruce Williams' paper describes a variety of these calculation "devices".

## Kant-en-Klaar Rekenaars en Tabulaire Calculators

Voordat de goedkope en gemakkelijk te gebruiken elektronische rekenmachines op de markt verschenen, waren de eisen, die gesteld werden aan commerciële berekeningen, te zwaar voor rekenlinialen, mechanische rekenmachines of logaritme-tabellen, met name als er sprake was van niet-decimale berekeningen. Een belangrijk antwoord op deze moeilijkheden vormden de Kant-en-Klaar Rekenaars en de Tabulaire Calculators, die gebruik maakten van allerlei voorbereide rekenresultaten. Het artikel van Bruce Williams beschrijft een aantal van deze onbekende rekenhulpmiddelen.

## Henny C. Brouwer

Ir. Henny C. Brouwer has a MSc in Architecture and Architectural and Urban Conservation from Delft University of Technology. Today she works as lecturer Architectural Conservation in Delft and as senior architect for the Government Buildings Agency of the Ministry of Housing, Spatial Planning and the Environment in The Hague, specialised in architectural monuments.

## Dutch Archtecture during the Initial Period of the Slide Rule



The slide rule was invented during the second decade of the $17^{\text {th }}$ century, the start of a period of enormous technological and scientific development. And for application of the new scientific insights slide rules and other mathematical instruments were needed.
The use of new technology by Dutch merchants made them very rich. With the money they earned the merchants and bankers built beautiful houses in the Netherlands, in particular in Amsterdam. The article in particular describes how mathematical ideas are used in the design of houses for the wealthy, placed in an extensive story of the historical and cultural backgrounds of the construction of these remarkable buildings.

## Nederlandse Architectuur in de Beginperiode van de Rekenliniaal

De rekenliniaal werd uitgevonden in de tweede decade van de $17^{\text {de }}$ eeuw, het begin van een periode van enorme technologische en wetenschappelijke ontwikkelingen. En voor de toepassing van de nieuwe wetenschappelijke inzichten waren rekenlinialen en andere mathematische instrumenten onontbeerlijk.
Het gebruik van nieuwe technologieën door Nederlandse handelaren maakte hen erg rijk. Met het verdiende geld bouwden de handelaren en bankiers prachtige huizen in Nederland, met name in Amsterdam. In het bijzonder beschrijft het artikel hoe wiskundige ideeën werden toegepast bij het ontwerpen van huizen voor de welvarenden, dit geplaatst in een uitgebreide historische en culturele context.

## Gerard van Gelswijck

After an early start as a turner at one of the biggest Dutch engineering firms Gerard van Gelswijck became, after fulfilling his duty in the National Service as a motorvehicle-mechanic, a mechanical engineer. He was for about 10 years with the firm where he started. Then he worked for 5 years at The Institute for Nuclear Physics Research. The Dutch Broadcast Organisation NOB became his next employer. There he has
 had various jobs, mostly concerning manufacturing and installation audio and video equipment in studio's and OB-vans. During the big turn-around and shake-out period of the early nineties he was the last remaining mechanical engineer. He has had different tasks ranging from managing the heating and electrical departments, running the autopool and the garage, and sorting-out the archives of the building department.

At the age of 55 he became redundant. Not for long because there was a shortage of teachers in technical colleges. Soon he had another job. Now he works voluntary in several organisations.

In the early part of his career he used, like everybody, a slide rule. Over the years he was given some sliderules by friends or colleagues but he was not a real collector. This attitude changed after meeting IJzebrand Schuitema. During the research for one of his books IJzebrand came in contact with Gerard and told him about the Kring, the Dutch Circle of Slide Rule Collectors. And so he started collecting slide rules besides collecting mostly Russian photographic equipment. Until now, in collecting slide rules he has no specific field, but his collection has a slight military bias.

## Slide Rules for Metal Workshops

In many branches of engineering slide rules were applied, in particular in the metal workshops. The article describes the art of cutting metal, the application of machine tools, milling and shaping machines. It explains the use of the Nestler 0260-Mechanica slide rule and slide charts in calculations for machine settings.

## Rekenlinialen voor Metaalwerkplaatsen

In veel technische bedrijfstakken werden rekenlinialen toegepast, in het bijzonder in metaalwerkplaatsen. Het artikel beschrijft de kunst van het metaalbewerken en de toepassing van machinewerktuigen en andere werktuigbouwkundige machines. Het laat zien hoe de Nestler 0260-Mechanica rekenliniaal en schuifrekenkaarten werden gebruikt bij het bepalen van de instellingen van die machines en werktuigen.

## Pierre Vander Meulen

Pierre vander Meulen was born April 30, 1945.
Degree: Ingénieur civil des constructions, graduated from Université Catholique de Louvain (Belgium) in 1969 and MBA from Université Libre de Bruxelles in 1990. Actual activities : Civil and Structural Engineer with Tractebel Gas Engineering (international Engineering active in liquefied gas storage and transport business). Manager of Civil and Structural Design department and specialized in the prestressed/steel storage tanks for LNG and LPG of very large capacity, up to
 $150000 \mathrm{~m}^{3}$.
Slide rule collector since 1990, owing a large number of Chinese slide rules. Is also interested in all calculating means.

## Reinforced Concrete Slide Rules

Pierre's article aims to supply an overall survey of the Reinforced Concrete Slide Rules (Rcsr's), which were produced during the slide rule era. It gives an introduction to the theory that is needed to understand the basics of reinforced concrete. With these basics in mind the reader can understand the use of the special slide rules of various designs for reinforced concrete

## Rekenlinialen voor Gewapend Beton

Het artikel van Pierre geeft een overzicht van de verschillende typen rekenlinialen die werden gebruikt door ingenieurs die met gewapend beton werkten gedurende de periode van de rekenliniaal. Het geeft een inleiding in de noodzakelijke theoretische kennis over gewapend beton. Met de theorie in het achterhoofd kan de lezer begrijpen hoe de verschillende typen rekenlinialen voor gewapend beton werden toegepast.

## Edwin J. Chamberlain

Edwin J. Chamberlain (Ed) is a semi-retired arctic engineer, specializing in the effects of frost action on the engineering properties of soils. He has studied in the US and Canada, and his work has taken him to the arctic regions of the world. His interests in slide rules date back to the 1950s when he was a civil engineering student in the US. He specializes in collecting long-scale and decimal-keeping slide rules. This paper reflects his interest in long-scale slide rules, of which he has about 50 in his collection - the longest being a $15-\mathrm{m}$ Loga rechenwalze. This paper updates an earlier report published in the Journal of the Oughtred Society.


## The Quest for greater Precision: Long Scale Slide Rules

Scales that are longer than the C- and D-scales on a 25 cm slide rule make calculations possible with a far greater precision than usual. The article reviews the the historical development of slide rules with long scales. It defines different categories and formats and it gives many examples of these long scale slide rules

## De Zoektocht naar grotere nauwkeurigheid: Rekenlinialen met lange schalen.

Schalen die langer zijn dan de gebruikelijke C- en D-schalen op 25 cm rekenlinialen maken veel nauwkeuriger berekeningen mogelijk. Het artikel geeft een overzicht van de geschiedkundige ontwikkeling van rekenlinialen met lange schalen. Het definiëert de verschillende categoriën en formaten en daarnaast beschrijft het vele voorbeelden van de betreffende rekenlinialen.

## Jörn Lütjens

Dr. Jörn Lütjens is lecturer at the Teacher Training College in Hamburg, Germany. From 1985 to 1990 he lived with his family in South Korea and was employed as an expert for vocational education at the Korea Institute of Technology. As a result of his special interest in Asian culture he has a small collection of abacuses, which consists of 84 items at the moment. Further collector activities are focused on slide rules and addiators. You can see it at his online-museum: www.joernluetjens.de


## The Abacus: one of the oldest Calculation Devices

Long before slide rules were used for calculations the abacus was the most important calculating device. The slide rule was used during a period of 350 years, but the abacus has been used for more than 2000 years. And even nowadays one can see the abacus in use in Russia and some Asian countries. The article gives an overview of the history of the abacus, the various systems and special constructions of the abacus. It also gives a great number of calculating examples by which the reader can understand how the abacus is used for additions, subtractions, multiplications and divisions, but also for calculating the square root of a number

## De Abacus: een van de oudste Rekenapparaten

Lang voordat rekenlinialen werden gebruikt bij berekeningen, was de abacus het meest toegepaste hulpmiddel bij het rekenen. De rekenliniaal was ongeveer 350 jaar in gebruik, maar de abacus meer dan 2000. En zelfs vandaag kan men hier en daar in Rusland en Aziatische landen de abacus nog gebruikt zien worden. Het artikel geeft een overzicht van de geschiedenis van de abacus, de verschillende systemen die werden toegepast, alsmede speciale constructies van abacussen. Het geeft bovendien een groot aantal rekenvoorbeelden waardoor de lezer kan begrijpen hoe de abacus werd gebruikt bij het uitvoeren van optellingen, aftrekkingen, vermenigvuldigingen en delingen, maar ook bij het trekken van de vierkantswortel uit een getal

# Gunter Rules in Navigation 

## Otto van Poelje

## Introduction

The Gunter rule is a legendary and much sought-after object for collectors of slide rules. Legendary, because of its naming after the inventor of the logarithmic scale (Edmund Gunter) and for its usage with chart dividers or compasses, but also mysterious because of its many exotic scales. Some slide rule collectors already have given their attention to the Gunter rule in references [22] to [25].
This article will further address the Gunter rule, with its background, its scales, and its application in navigation at sea. Also some other navigational rules from that period will be described.

## Navigation in Gunter's time

Early $17^{\text {th }}$ century was a turning point for navigation, from "plane sailing" on sea charts with rectangular grids, to the use of Mercator charts for plotting compass courses as straight lines (the so-called "loxodromes"), see [20]. In mathematics, the field of trigonometry and spherical goniometry was already well-known. By astronomical observations of the altitude of sun or Polar star with the cross-staff (the precursor of octant and sextant), the latitude of a ship's position could be found. The longitude was a much more difficult problem, only to be solved later by accurate chronometers, better calculation methods ("Line of Position"), and eventually by wireless technology like Decca, LORAN and GPS. The challenge for the sailors of that time was to introduce the new techniques in their navigation methods. The practice of solving navigational problems by construction of course lines on the sea charts was to be supplemented by the newly invented calculation methods by logarithms.

## Definition

When is a rule a Gunter rule?
As always, more than one answer is possible, but the main criteria appear to be that it is a fixed rule, without moving parts, and that it has one or more logarithmic scales. The inherent assumption being that with these characteristics it is possible to multiply or divide by moving scale distances between the tips of the dividers. This is the most generic type of definition for a Gunter rule.
Now there exists one specific type of Gunter rule which collectors encounter very often in their search for new acquisitions. In my own contacts with fellow collectors about Gunter rules up till now, I know already of some hundred specimens of that type. They also can be found in many museum displays on slide rules. This specific type could be described in pages full of definition text, but the easiest definition is by graphical example: the best picture of the most often encountered Gunter rule can be found as a fold-out drawing in [22], drawn by Bruce Babcock. This we will call the "Standard Gunter Rule". It has an amazing large number of scales: 22 in total, including a "diagonal scale" for determining exact line lengths by means of dividers.
The Standard Gunter Rule is most often constructed of wood, but sometimes of brass or German silver, and also 1foot specimens of ivory have been sighted.
The majority of known Gunter rules do not bear a Maker's name or date.
Of course variations are encountered, like scales with different name abbreviations, extended scales (like the "Donn"-variant with square, cube and military scales, see [14]), but also different sizes. Most Gunters are 2 feet by 2 or 1.5 inches. Also 1 -foot models exist, with the same Standard Gunter Rule scales compressed in this smaller size. Still the Standard Gunter Rule definition covers an amazing large proportion of all known Gunter rules.


## History of calculation by logarithms

Throughout history there has always been an ever-increasing need for calculations. For example, at the end of the $16^{\text {th }}$ century, Kepler had published his laws on planetary motion, and this knowledge resulted in the need for massive calculations to determine the orbits of our planets. But the art of multiplication and division was tedious handwork, only known to professional "calculators".
An approach existed, already then, to transform the operands x and y of a multiplication, into a goniometric domain with the formula

$$
x \cdot y=\sin (a) \cdot \sin (b)=\{\cos (a-b)-\cos (a+b)\} / 2
$$

where $\mathrm{a}=\arcsin (\mathrm{x})$ and $\mathrm{y}=\arcsin (\mathrm{y})$. This allowed the multiplication to be replaced by plus and minus operations in combination with sine and cosine tables (which were available at that time) for the necessary transformations.
The search for logarithms had the objective to find a simpler transform:

$$
\log (x . y)=\log (x)+\log (y)
$$

Three men of science played a major role in the history of the Gunter rule:

- John Napier (1550-1617), the first to publish the concept of logarithms and their corresponding tables in 1614
- Henri Briggs (1556-1630), calculator and author of the first decimal-based logarithmic table in 1617
- Edmund Gunter (1581-1626), inventor of the logarithmic scales (published in 1624)

These men had all the characteristics of scientists of that time: they could read, write, and probably converse, in Latin. They had a very wide and universal range of interests (unlike today's specialised scientist). For example, Gunter was active in theology, surveying, astronomy, navigation, sundial design and even the timedependency of the earth's magnetic field variation. He was a more decimaloriented person than today's average Englishman, because he divided his own surveying unit, the "Gunter Chain" of 22 yards, in 100 "Links"; he also proposed to divide each of the 360 degrees of a circle into decimal fractions, in stead of sexagesimal minutes and seconds.
Then their speed in publishing was not our "internet speed": it could take many, many years before study resulted in publication, which even then was limited to a Latin-reading public.
John Napier worked for almost 20 years on his concept and tables of the logarithms before publishing the results in his famous "Mirifici Logarithmorum Canonis Descriptio" (Description of the Miraculous List of Logarithms), see [1]. His logarithm, here called LN, would be written in today's notation as

$$
\mathrm{LN}(\mathrm{x})=-10.000 .000 \ln (\sin (\mathrm{x}))
$$

The factor of "ten million" was related to the fact that integer numbers were favoured in tables (because decimal fractions, though already known in principle, were not common knowledge and even had various different and conflicting notations).
The natural logarithm ("In") in this formula turns out to be the consequence of Napier's numerical approach for calculating logarithms. He described his logarithms in a kinematical model, and not in terms of the power of a number.

Only a century later the mathematical context would be created for the "ln (x)" as an analytical function, with
Figure 2: Standard Gunter Rule Euler's constant $\mathrm{e}=2.718 \ldots$ as base number.

It is remarkable that Napier's tables only addressed the logarithms of sines, and not the logarithms of numbers. Napier assumedly gave priority to goniometric calculations as needed in astronomy and navigation, before his poor health prevented him to calculate other tables.
Henri Briggs, a Professor of Geometry at Gresham College in London for already many years, was deeply impressed by Napier's publication in 1614, and he travelled two times to Edinburgh for lengthy discussions with Napier on the continuation of his research. They agreed to get rid of the factor of "ten million" (resulting in tables containing decimal fractions with Napier's "dot" notation), and to use the base of 10 for future log tables, which Briggs took it upon himself to calculate.
Soon after Napier's death in 1617, Briggs published his first decimal log table [2], from 1 to 1000 (the first "Chiliad", in his classical terminology). Later the tables were extended by himself and others.
Still, Briggs only published logarithms of numbers, and not of goniometric functions like Napier had done. This means that a formula like

$$
X / \sin (x)=Y / \sin (y)
$$

could not have been calculated directly using Napier's tables, nor by Briggs' tables.
And that calculation happens to be one of the most important formulae of plain navigation at sea, the Rule of Sines.

## Enter Edmund Gunter

When Gresham College needed a new Professor of Astronomy in 1619 , Briggs recommended his friend Gunter for the vacancy. The two of them must have had close interaction on the subject of logarithms. Unlike an ivory-towered university, Gresham College held public lectures and disputes in the English language, and Gunter must have received a lot of practical feedback from ship captains and other navigators in his audience.
A number of his known accomplishments are related to navigation at sea: he invented for example the log line to measure a ship's speed (with "knots" for direct reading of the speed measured), and Gunter's quadrant, an astronomical altitude measurement and calculation device, related to the older astrolabium .
Gunter must have realized that navigational calculations would benefit from logarithms of both numbers and goniometric functions. In 1619 he published the first combined table [3], the "Canon Triangulorum", containing his own -newly calculated- decimal logarithms of sines and tangents, but he also added Briggs' logarithms of numbers. Now, at last, the Rule of Sines could be calculated by logarithms from tables in a single book.
Still, the goniometric tables in Gunter's book look strange to us, because he had added the term 10 to every entry in order to prevent negative numbers which were not fashionable in his time. For example, $\log (\sin (30))=-0.30103$, but in Gunter's table it was uplifted to 9.69897 .


Figure 3: Page from Canon Triangulorum

For general use, this would cause an incompatibility with Briggs' tables, but in Gunter's calculation examples the term 10 was always cancelled out by the fact that only ratios of sines were involved.

## Gunter's line and Gunter's scales

Gunter took the matter one step further. He must have felt that some navigational problems needed an easier and faster solution than calculation by tables could provide, for example in coastal water navigation, in "haven-finding" situations.
As can be seen in Gunter's book [4] on various navigational instruments (1624), he was well aware of the "crossstaff" and the "sector" (in other languages called "proportional" instruments), and of the efficient use of their many already existing- scales. The use of dividers for making calculations on a sector was a practice already known in the $16^{\text {th }}$ century (the invention of the sector is sometimes attributed to Galileo, but not conclusively).


From there, it must have been a logical step for him to design a new type of scale, where numbers were represented by logarithmic scale distances, and where dividers were used to add or subtract those distances in the logarithmic domain. He called his scale the "Line of Numbers", other people used the term "Line of Proportion" or "Gunter's Line". His scale should not be confused with the "Line of Lines", the fundamental linear scale on a sector.

Actually, Gunter proposed in [4] three types of logarithmic scales, not only the Line of Numbers but also the logarithmic sine and tangent scales, so that the Rule of Sines could be calculated between these scales; he also hinted that the addition of a "versed sine" scale might be easier for calculating the sides of a spherical triangle.

Gunter called any logarithmic scale "Artificial", because that was the term which Napier used originally, before he replaced it with the term "Logarithms".

Figure 4: Sector with dividers


Figure 5: Original drawing of Gunter's Scales

The power of Gunter's concept is proven by the fact that his 2-cycle Line of Numbers and the corresponding SIN scale have maintained exactly the same structure as the A/B-scale and the S-scale of industrially-produced slide rules until early $20^{\text {th }}$ century.
These Gunter scales were proposed and described in the chapter "on the Cross-staff", as it appeared that this navigational instrument for measuring altitude of sun or stars still had space available for additional scales. Gunter did not mention his artificial scales in the chapter "on the Sector", and we don't know the reason for that: maybe he found one description was enough, the one in the Cross-staff chapter, or might he have had other reasons? However, none of the known cross-staffs reported in [21] have Gunter's lines or scales engraved at all.


Figure 6: Gunter's scales on the English Sector

On the other hand, an "English" variant of the sector is known, also called Gunter's Sector (although it is not mentioned in the original chapter "on the Sector" in Gunter's book), on which Gunter's three basic scales (marked as $\mathrm{T}, \mathrm{S}, \mathrm{N}$ ) are available at the bottom of one side, when the instrument is opened.

This version has been produced till the end of the $19^{\text {th }}$ century, and can quite often be found at fairs or auctions, mostly in ivory.
Gunter's publications were so popular that they were reprinted many times, even after he died in 1626.
In the 1673 reprint [10] many appendices had been added, for example one after the chapter "on the Sector", by Mr. Samuel Foster: "The Sector Altered; and other Scales added: with the Description and Use thereof". In this additional chapter (page 161) the addition of the three Gunter lines is mentioned: "The Sector then being opened and so made a streight Rular; the outer edge hath inscribed upon it the three usual Scales of Logarithmetical Numbers, Sines and Tangents".

This looks very much like the English sectors that collectors are still finding today. In this supplemented re-print, the Standard Gunter Rule could have been mentioned and described, if it already


Figure 7: The 1673 edition of Gunter's Works existed by then. But it is not there.
The fact that usage of Gunter's basic 3 scales on a straight and plain rule was not mentioned in the 1673 additions, might suggest that the Standard Gunter Rule was introduced only later, end $17^{\text {th }}$ century or even $18^{\text {th }}$ century. We must consider that changes happened slower in those days, and especially sailors were reputedly slow -see also [16]- in the acceptance of "high-tech" innovations, like the Gunter scales must have been.
It is very well possible that someone else than Gunter had realized the possibilities of these new scales on the surface of a plain 2-feet rule. The advantage would be that there is ample space for many scales on the two sides of a 2 inch wide rule, with the increased accuracy of 2feet scale length. The absence of moving parts would be an additional advantage in a ship's rough environment, and a long rule is useful anyway on a chart table.
This would mean that the Gunter Rule is named after Gunter only because his Line of Numbers and his artificial SIN and TAN scales are included, not because he invented the rule "in toto".
In that case, the still remaining question is, who then did design the Gunter Rule, using the scales in Gunter's book, and when? Any contribution to answering this question would be greatly appreciated.

## The scales on a Gunter rule

When we look at the Standard Gunter Rule, we see two faces containing the following information.
Front: a "diagonal table" on the left side, and a number of mainly goniometrical scales on the right side (none of which are logarithmic).
Back: a set of full-length scales, most of which are of a logarithmic ("Artificial") nature.
Many of these scales have been described extensively in [11] with respect to construction and usage.
The next table gives a summary of name and meaning of the main scales (the "formula" column gives the proportional length on a scale for a number marked X).

## Front

| $\begin{array}{c}\text { Scale } \\ \text { Abbrev. }\end{array}$ | Full Name | Meaning | Formula |
| :--- | :--- | :--- | :--- |
|  | Diagonal Scale on left side | To set exact lengths with dividers |  |
|  | Inches | Measurement scale along the side |  |
| L E A | Leagues | $\begin{array}{l}\text { Linear scale: } \\ 1 \text { league }=3 \text { sea miles }\end{array}$ | X |
| R U M | Chords of Rhumbs | $\begin{array}{l}\text { Chord is twice the half-sine, for } \\ \text { compass points }\left(32 \text { in } 360^{\circ}\right)\end{array}$ | $2 \sin (5.625 \mathrm{X})$ |
| C H O | Chords of Degrees | $\begin{array}{l}\text { Chord is twice the half-sine, } \\ \text { for degrees }\left(360^{\circ}\right)\end{array}$ | $2 \sin (\mathrm{X} / 2)$ |
| S I N | Sine of Degrees | Sin for degrees $\left(360^{\circ}\right)$ | $\sin (\mathrm{X})$ |
| T A N | Tangent of Degrees | Tan for degrees $\left(360^{\circ}\right)$ | $\tan (\mathrm{X})$ |
| S * T | Semi - Tangent | Half-Tan for degrees (360 $)$ | $\tan (\mathrm{X} / 2)$ |
| $\begin{array}{l}\text { M } ~\end{array}$ L |  |  |  |
| or |  |  |  |
| L O N |  |  |  |\(\left.~ $$
\begin{array}{ll}\text { Miles of Longitude } & \text { Length of 1 degree at longitude X }\end{array}
$$ \begin{array}{l}60 \cos (\mathrm{X}) , to be <br>

combined with <br>
CHO/RUM scale\end{array}\right]\)

## Back

| Scale <br> Abbrev. | Full Name | Formula |  |
| :--- | :--- | :--- | :--- |
| S * R | (Artificial) Sine of Rhumbs | Logsin for compass points | $\log (\sin (11.25 \mathrm{X}))$ |
| $\mathrm{T}^{*}$ R | (Artificial) Tangens of Rhumbs | Logtan for compass points | $\log (\tan (11.25 \mathrm{X}))$ |
| N U M | (Artificial) Line of Numbers | 2-cycle Log Scale (like the A or B <br> scale of a modern slide rule) | $\log (\mathrm{X})$ |
| S I N | (Artificial) Sine of Degrees | Logsin for degrees $\left(360^{\circ}\right)$ | $\log (\sin (\mathrm{X}))$ |
| $\mathrm{V}^{*}$ S | (Artificial) VerSine of Degrees | Logversin for degrees $\left(360^{\circ}\right)$ | $\log (1-\cos (\mathrm{X}))$ |
| T A N | (Artificial) Tangent of Degrees | Logtan for degrees $\left(360^{\circ}\right)$ | $\log (\tan (\mathrm{X}))$ |
| M E R | Meridional Line | Length of 1 degree latitude on a <br> meridian of the Mercator map | $\sec (\mathrm{X})$ |
| E * P | Equal Parts | Linear scale | X |

## Use with dividers or compasses

One cycle on the NUM scale of the Standard Gunter Rule has a length of $11 \frac{1}{4}$ inches. To fit distances up to that value between the tips of the dividers, it would require dividers of at least 8 " (for a maximum opening angle of some $90^{\circ}$ ). Sea chart dividers had that size, and were sometimes even larger.

## Some observations on the scales

The Gunter Rule has different scale characteristics compared with a normal slide rule because of its use with dividers. The modern slide rule has its scales vertically aligned for usage with cursor and vertical hairline. Even on the sector the scales were aligned to the rotation point.
On the Gunter Rule the scales can be positioned anywhere as long as the dividers can measure a length on a scale. Therefore one sees sometimes up to three different scales adjoined on one horizontal line.
There are some exceptions, for example the $\mathrm{M}^{*} \mathrm{~L}$ and CHO are vertically aligned because the number of Nautical Miles in one degree of a parallel are read on $M^{*}$ L against the latitude of that parallel on the CHO scale (the result $\mathrm{M}^{*} \mathrm{~L}$ ranges from 0 on a pole to 60 Nautical Miles on the equator).
The length of different scales have to correspond of course: for example all the logarithmic scales on the back side can be used against one another, to allow any multiplication or division between numbers and goniometrical functions.
As a practical example, reference
[23] relates the solution by Gunter rule of a coastal navigation problem from [14].

## Example from Bowditch

From the 1851 edition of Bowditch, [13], the most simple

Course and distance sailed given, to find the difference of latitude and departure from the meridian.
A ship from the latitude of $49^{\circ} 57^{\prime}$ N., sails S. W. by W. 244 miles; required the latitude she isin, and her departure from the meridian sailed from.

## BY PROJECTION.

Draw the line CA, to represent the meridian of the place $\mathbf{C}$, from whence the ship sailed. With the chord of $60^{\circ}$ in your compasses, and one foot in C, as a centre, describe the compass W. S. E. Take 5 points in your compasses from the line of rhumbs on the plane scale, and set it off on the arc, from $S$. towards 'W., for the course; through this point and C draw the line CB , and make it equal to the distance 244; draw BA parallel to the east and west line EW, to cut the meridian in A. Then will CA be the difference of latitude 135.6, and $A B$ the departure 202.9.


## BY LOGARITHMS.

By making the distance radius.

| To find the departure. |
| :--- |
| As radius 8 points.......... |

To find the difference of latitude.
As radius 8 points. . . . . . . . . . . 10.00000
Is to the distance 244 .......... 2.38739
So is the cosine course 5 points. 9.74474
To the difference of lat. 135.6.. $\overline{2.13213}$

Now, as the ship is in north latitude sailing southerly,

> From the latitude left $. \ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . .$.
> Take the difference of latitude $185.6 . \ldots . . . . . . . . . . . . .$.
> Gives the latitude in...................................... $\overline{47 \cdot 41} \mathrm{~N}$.
> And the departure from the meridian is 202.9 miles.

BY GUNTER.
Extend from radius or 8 points* to 5 points on the line marked SR ; that extent will reach from the distance 244 , to the departure 202.9 , on the line of numbers.

2dly. Extend from radius or 8 points to 3 points, the complement of the course, on he line SR ; that extent will reach from the distance 244 , to the difference of latitude 135.6 , on the line of numbers.

Thus may all the operations be performed in the several cases of Navigation.
example will be shown for "plane sailing", i.e. not yet taking into account the effects of a Mercator type chart. The three possible methods of solution of the problem are given: the construction by projection, calculation by logarithm tables, and solution by Gunter's scales.

## Accuracy of scales

In principle, the large size of the Gunter Rule would allow for good accuracy, but the production methods used resulted in an accuracy much worse than, say, of a large desktop Nestler.
In the first place the makers economised on the scale divisions: on the NUM scale some 130 division marks were made, while a modern 50 cm A-scale can have three times as many. On the SIN scale the distance between two marks can be more than 1 cm , which makes visual interpolation difficult.
In the second place, the divisions were made by hand, before the introduction of the "dividing" engine. This added to the inaccuracy: it has been reported, see page 29 in [25], that Gunter rules could be engraved by hand with a speed of about 1 mark per second. This would bring the total mark engraving time of a Standard Gunter Rule, with more than 1200 marks in all, to a full 20 minutes of concentrated and error-prone work.
Lastly, the scale designs were being copied and copied again, sometimes including mistakes. For example, my own collection includes a particular 1-foot Gunter Rule where the RUM and CHO scales differ more than $5 \%$ in length!
On the other hand, the mariner did not always need the highest accuracy. He measured his course not in degrees but in compass points, and that was the best accuracy with which he could steer under the conditions of strong waves, or inaccurate magnetic deviation and variation.
Some of the most frequent calculations made, were those in "dead reckoning" (strange name, derived from
"deduced" reckoning), where a new position was deduced from a previous position in a "course triangle" consisting of speed vectors for course steered, side effect of current and wind, and the true course as result. This deduction was either done by construction with compasses on chart or paper, or by calculation with the Gunter rule on NUM and SIN/TAN scales (again this Rule of Sines).

## CHO

One of the popular scales was the chord scale, which could be used for measuring or constructing angles. The chord " k " of an angle a (AMB) is the line that connects A and $B$, the points of intersection between the legs of angle $a$ and the circle. The length can be expressed as a goniometrical function: $\operatorname{Chord}(a)=2 * r * \sin (a / 2)$. This explains the old term (double) "halfsine" for the chord function.
An angle of $60^{\circ}$ has a chord equal to the radius $r$.
To construct a given angle, first the radius $r$ is taken with compasses from the chord


Figure 9: Rhumb compass points scale at $60^{\circ}$ (frequently used values like this one have a brass inlay on


Figure 8: Chord construction the scale to fixate the points of the compasses and to protect the surface of the rule). Then the circle with this radius is drawn. Finally the chord length of the required angle is copied from the chord scale to the circle drawn.

## RUM

Mariners in Gunter's time already used the magnetic compass with an angular scale called "rosa".
The compass rose was (and still is) divided in 32 compass points, also called rhumbs. While the main compass points are North -East - South - West, the intermediate compass points have logical names like North-West, East-North-East etc.
One rhumb is equivalent to $360 / 32=11.25$ degrees.
Some goniometric functions like chords and artificial sines and tangents had separate scales on the Standard Gunter Rule, one for degrees and one for rhumbs, thereby proving its maritime nature. Those double scales were vertically aligned, with 8 compass points over the $90^{\circ}$ mark.
The scale called RUM is actually the chord scale, but for rhumb compass points in stead of degrees.

The Standard Gunter Rule even has two sets of CHO/RUM scales, for different radii (27/8" and $41 / 4 "$ ).

## Other Rules for Navigation

The Standard Gunter Rule, although widely known to collectors, was certainly not the only rule with scales designed for navigation.
Around the same time that Gunter published his work on the cross-staff etc, a small book, see [6], was published by John Aspley, titled " Speculum Nauticum ...". This book explains the use of a so-called "Plain-Scale", a rule with only 5 scales, which are the equivalents of the Gunter scales RUM, CHO , E*P on the front, and NUM and $\mathrm{M} * \mathrm{~L}$ on the back.

The Plain-Scale or Plane-Scale, was the general name of a navigation rule designed to project spherical problems onto the 2 -dimensional plane of chart or drawing.
Some, but not all, Plains-Scales belong to the general class of Gunter rules, when at least the NUM scale is included.

Aspley only gives a picture of the front side, but in reference [16] the Dutch expert on the history of navigation, Ernst Crone, supplies a picture of front and back of the Plain-Scale. This more complete picture has been found in a Dutch handbook for navigators [8] which addresses the same subject as Aspley's book, though without any reference to him. But even Aspley may not have been the original Plain-Scale inventor, who must have been well aware of NUM, Gunter's "Line of Numbers". Remarkable about this Plain-Scale picture is that the 2-cycle logarithmical scale on the back is an inverse scale, and that the $\mathrm{M}^{*} \mathrm{~L}$ scale is different in that it is divided into 15 units of 4 sea miles each ("German" miles); in Aspley's version the $\mathrm{M}^{*} \mathrm{~L}$ scale is divided into 20 leagues (of 3 sea miles each).

Another "Plain-Scale" has been described in [9], one of the Dutch navigation handbooks of the mid-17 ${ }^{\text {th }}$ century.


Abraham de Graef describes a somewhat extended Plain-Scale (one-sided) where the following table translates his scale indications into our usual Gunter scale names.

## Actual usage of Gunter Rules or Plain-Scales

Looking in navigation handbooks, we can gain some insight into which type of navigation rule was used, by country. Obviously, the Gunter rule was used in England, but also in the United States because it is mentioned far into the $19^{\text {th }}$ century in the handbook [13] of Bowditch: "The New American Practical Navigator". Also in Germany, there is at least one navigation handbook [14], by Jerrmann, 1888, describing the use of the Gunter rule.
In Holland, the Gunter rule may have been used less, or maybe not at all. Edmund Wingate had published in 1624 a book, [5] \& [15], describing Gunter's scales (while giving him proper credit), not long after Gunter's

| de Graef‘s <br> scale names | Gunter‘s <br> scale names |
| :--- | :--- |
| Sinus | S I N (non-log) |
| Tangens | T A N (non-log) |
| B | R U M |
| D | C H O |
| F | M * L |
| M | T A N (non-log) |
|  | E * P |
|  | N U M |

Figure 11 : de Graef's Plain-Scale
own publication. This book was translated into Dutch, see [7], including a fold-out picture of the three Gunter scales. Consequently, a description of Gunter's scales was easily available in the Netherlands, via this detour.

A number of Dutch navigation handbooks from the $17^{\text {th }}$ century have been browsed, but none of them mention the Standard Gunter Rule: only some simple versions of a Plain-Scale are described.
But then, one would expect more of these Plain-Scales preserved or recovered, just like so many Standard Gunter Rules have survived. In a series of articles [17] and [18], a description is given of one Plain-Scale that was found in the Dutch East Indiaman sailing vessel "Hollandia", which was wrecked in 1743 off the Scilly Isles.

That particular Plain-Scale had again a different design, with scales on one side and a diagonal table on the other. The table on the left gives the relation to our usual Gunter scale names.

| Hollandia <br> scale names | Gunter's <br> scale names |
| :--- | :--- |
| V | Chord scale in <br> special compass <br> points (24 per $360^{\circ}$ ) |
| H | S I N (non- <br> logarithmic) |
| S | R U M |
| C | C H O |
| L N | N U M |
| V | V (smaller radius) |
| S | S (smaller radius) |



Figure 12: The Hollandia Plain-Scale

The Hollandia Plain-Scale is signed by Johannes van Keulen (JVK), a well-known cartographer and instrument maker for the VOC ("Verenigde Oost-Indische Compagnie", or Dutch United East India Company). This object is part of the collection in the "Rijksmuseum" in Amsterdam. Only three other specimens of a similar Plain-Scale are known: one in the "Nederlands Scheepvaartmuseum" in Amsterdam, signed by Jacobus Kley, one in the "Zeeuws maritiem muZEEum" in Vlissingen, and the last one is owned by the "Museum Boerhaave" in Leiden.
Another source of information consists of actual lists of navigation equipment supplied. In [19] a list is discussed in which the standard navigating equipment for the VOC ships is named, with prices and numbers used per ship. In 1747, three "Pleinschalen" (Plain-Scales) were required per ship, at a price of 12 "Stuyvers" (or 0.60 gulden) each: this may be compared with the more than 100 -fold price of a modern Hadley octant ( 75 gulden). Also in other equipment lists only "Pleinschalen" are mentioned, and not Gunter rules. On the other hand, Gunter rules were actually mentioned in catalogues of Dutch instrument makers, for example Johannes van Keulen lists in 1777 a Gunter rule for 1.50 gulden.


## Conclusions

The Gunter rules, that most collectors know and seek, are so identical to each other that we call them Standard Gunter Rules. There are variations, sometimes with small differences, but in the Netherlands some versions of the general Plain-Scale appear to have had preference over the Standard Gunter Rule. Gunter rules have by definition one or more logarithmic scales. A number of other PlainScales are known without any logarithmic scale at all, like this one from Bowditch, see [13],

Figure 13: Plain-Scale without Gunter's lines

1851, Plate II before page 17.
The non-logarithmic scales on Gunter rules and Plain-Scales are mostly derived from earlier instruments, like the sector and the cross-staff.
It appears that Gunter was not the designer of the complete Standard Gunter Rule, although he is conclusively considered to be the inventor of the three basic logarithmic scales NUM, SIN and TAN.
This is emphasized by Oughtred himself, when he writes modestly about his own Circles of Proportion, see [12]:
"For these, I must freely confess, I have not so good a claim against all men, as for my Horizontal instrument. The honour of the invention [of logarithms] next to the lord of Merchiston, and our master Briggs, belonging (if I have not been wrongly informed) to master Gunter, who exposed their numbers onto a streight line. And what doth this new instrument, called the Circles of proportion, but only bowe and inflect master Gunter's line or rule."

Further study is needed to discover the real roots and the development line of the Standard Gunter Rule.

## Acknowledgments

Many fellow collectors have contributed to this paper with remarks, suggestions, or by providing information and pictures of their own Gunter rules.

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## Literature references, in chronological order:

[1] Napier, J., "Mirifici Logarithmorum Canonis Descriptio", 1614
[2] Briggs, H., "Logarithmorum Chilias Prima", 1617
[3] Gunter, E., "Canon Triangulorum", 1620
[4] Gunter, E., "The Description and Use of the Sector, the Crosse-Staffe and other instruments", 1624
[5] Wingate, E., "l'Usage de la Reigle de Proportion en l'Arithmetique \& Geometri", Paris, 1624
[6] Aspley, J., "Speculum Nauticum, A Looking Glasse for Seamen: Wherin they may behold a small Instrument called the Plain Scale ...", 1624
[7] Leemkulius, W., "des Evenrednigen Liniaals, Tel- en -Meetkunstig gebruyk", 1628
[8] Ruyter, Dierick, "De Platte, ofte PLEYN-SCHAEL verklaert", 1631
[9] de Graef, A., "Beschrijvinge van de nieuwe Pleynschael", 1658
[10] Gunter, E. a.o., "The Works of that Famous Mathematician Mr. Edmund Gunter", 1673
[11] Bion, M., Stone, E., "The Construction and Principal Uses of Mathematical Instruments", 1709. Facsimile reprint of the 1759 version by Astragal Press, NJ, USA, 1995; the 1759 edition is translated from French into English and supplemented greatly by E. Stone
[12] Ward, J., "The Lives of the Professors of Gresham College", London, 1740, p. 77-81
[13] Bowditch, N., "The New American Practical Navigator", 1802 through late $20^{\text {th }}$ century
[14] Jerrmann, L., "Die Gunterscale", Hamburg, 1888
[15] Cajori, F., "On the history of Gunter's scale and the slide rule during the seventeenth century", 1920. Facsimile reprint by Astragal Press, NJ, USA, 1994
[16] Crone, E., "De Pleinschaal", De Zee, p. 441-465, 572-592, 1927
[17] Cowan, R.S., "The Pleinschaal from the Hollandia", IJNA 11.4 p. 287 - 290, 1982 (International Journal of Nautical Archeology and Underwater Exploration)
[18] Engelsman, S.B., "The Navigational Ruler from the Hollandia" (1743), IJNA 11.4 p. 291-292, 1982
[19] Mörzer-Bruyns, W.F., "A History of the Use and Supply of the Pleynschael by Instrument Makers to the VOC", IJNA 11.4 p. 293-296, 1982
[20] Davids, C.A., "Zeewezen en Wetenschap, de wetenschap en de ontwikkeling van de navigatietechniek in Nederland tussen 1585 en 1815", Doctoral thesis, Amsterdam, 1984
[21] Mörzer-Bruyns, W.F., "The Cross-Staff, History and Use of a Navigational Instrument, Walburg Press, 1994
[22] Babcock, B.E., "Some Notes on the History and Use of Gunter's Scale", Journal of the Oughtred Society Vol. 3, No. 2, p. 14-20, 1994
[23] Jezierski, D. von, "Further Notes on the Operation of the Gunter Rule", Journal of the Oughtred Society Vol. 6, No. 2, p. 7-8, 1997
[24] Otnes, R., "The Gunter Rule", Journal of the Oughtred Society Vol. 8, No. 2, p. 6, 1999
[25] Jezierski, D. von, "Slide Rules, A Journey Through Three Centuries", pages 2-6, Astragal Press, NJ, USA, 2000


## DEN DANSKE REGNESTOK MED VERDENSRY



## History of UTO and other Danish Slide Rule Firms

## John Kvint

## The time before UTO

The oldest firm connected to slide-rules is Linex, founded by Frede Duelund Nielsen, a civil engineer in electrical engineering. During his study in the polytechnical school in Copenhagen, in 1922, he learned how to make simple drawing articles, and soon financed his study by selling curves and triangles to his fellow-students. Having different occupations in the following years, he continued his "hobby" of supplying the students with drawing articles. Wholesalers got interested, and he soon realized that he needed a dividing machine to make scales on rulers and triangles.
The first prototype of a dividing machine was succeeded by a more sophisticated model, and Duelund was awarded a technical prize in 1936, after which he decided to make his living by manufacturing drawing articles.

Linex was established 1936 in Rødovre, 10 km west of Copenhagen. The main material was celluloid, but after 1936 the far superior Plexiglas
 became available, and due to its pre-war import, Linex could import a limited amount of Plexiglas during the second world war.
However, Linex could not exist on the few triangles that could be made thereof, so production was changed to wristwatch glasses, and Linex soon became the main supplier to the watchmakers.

In nearby Gentofte, a merchant had established a general store in 1854, supplying the surrounding farms. Slowly a village grew around the store and in 1873 the merchant Ole Nielsen took over and expanded the shop to contain a lumberyard and a fuel store.
In 1910 the son, Olyuf Nielsen, took over and expanded further into a street of shops for tobacco, cameras, paint, and a cinema (which still exists). He also founded DIWA manufacturing waterlevels (the name is abbreviated from Danish Industri Waterpas A/S), and by the beginning of WWII DIWA was a major exporter of waterlevels.

In 1941 two young engineers, Kruuse and Larsen, being unemployed, founded Kruuse and Larsen
Instrumentfabrik, with the intention of manufacturing slide rules. From the beginning they had shortage of material but soon after they had the fortune to get in contact with DIWA, having big stocks of mahogany being seasoned, plenty of woodworking machinery, and in due time they had DIWA running a line of slide rules. Kruuse's and Larsen's main and major contribution was a self-developed dividing machine. Larsen also developed the main part of tools and machines necessary for the production of slide rules.

The wooden slide rules made by DIWA, were of the type Rietz, and later Darmstadt. DIWA got a very fine reputation, and was prime supplier to education in schools, universities and technical high schools.
After the war, DIWA changed to PVC and made both duplex and fullbodied slide rules.
The range was extended to the types Businessman and Electro.
The production employed 90 people in slide rule manufacture., and export was necessary to keep production ongoing. Some countries were not allowed foreign currency, so Nielsen decided to create a joint venture
 in India. Kruuse and Larsen provided all the machinery, but the Indian partners would not or could not pay, and DIWA was close to bankruptcy.

## Founding of UTO

Kruuse and Larsen left DIWA in 1957 and started their own manufacturing. They named it UTO, just to have a short practical name, that could be pronounced in most languages.

UTO from the beginning took up the photo-etching process, that allowed full-scale faces to be "printed" in one operation, instead of the engraving method used by DIWA.

UTO also had to rely mainly on export, as DIWA sat firmly on the domestic market. But the slide rules were extremely fine and precise due to the photo-etching, and UTO slide rules were soon to be considered an important alternative to the German slide rules.

Linex got the acrylic raw material again after the war, and took up drawing articles again. Being supplied from ICI with Perspex, and based on several years experience in processing acrylic material, Duelund developed and patented a Rietz slide rule of white Perspex having a transparent slide, allowing the reading of sines and tangents by just looking through the slide. I doubt it was ever commercialised.
In 1951 Linex got its first injection-moulding machine, and after some experience in that field, it was decided to make an injection-moulded slide rule. First (and only) model was a pocket slide rule.
Having only the scales [inch, A/B,CI, C/D, cm], it was full-face printed in hot-foil stamping. Variations hereof having only the importers name, or the name of the school, were exported world-wide. From the production numbers, it must be assumed that some countries imported one for every pupil of their schools. Instead of developing further into slide rules, Linex used their precision dividing machines to produce triangular and flat reduction scales. Making "engine divided" scales was a powerful sales point, and the export was growing.

UTO needed more space, and as the government offered cheap mortgages when settling in undeveloped corners of Denmark, Larsen decided in 1971 to move to Tønder, situated near the German border.
One of the important products was the pocket slide rule imprinted with the customers name.
He also took up the manufacturing of drawing articles, and became an important supplier to Linex with photo-etched products, especially spiral transition curves for road construction. UTO relied on

machining sheet material, and on dividing by photoetching.

In the nineteen sixties, the Copenhagen printing firm IWACO specialized in slide charts. Mainly made of thin cardboard, they filled a need of a generation, partly for promotion but also as guides for the rapidly developing new products after the war. For example: trouble-shooting of television sets or colour-coding of resistors. In 1978, a subsidiary CRETEX was founded to specialize in plastic foil-wrapped slide charts. The printing method was silk-screen using the most modern equipment developed for precision printing of electronic circuits. CRETEX grew during the following years, got a reputation of being just the supplier of slide charts, but never made their own - all were custom made. Later CRETEX expanded into acrylic sales displays, and importing advertising objects, like ashtrays having tobacco brand names. CRETEX was sold in 2000, and the new owner tries his luck in vacuum-forming of sales displays for cosmetics and the like.

DIWA developed further during the sixties and seventies. Log-log, Polylog and Teknilog kept the business going, but the cost of labour increased, and to be more cost-effective the sales were turned over to Linex, having a wordwide platform of national importers. Finally Diwa closed its activities in 1979, but the sales continued as a large stock had been built up. The DIWA machinery was partly taken over by Linex, but not fully utilized, as Linex had switched to robot manufacturing and sheet materials machining by CNC-machines. One thing happened: The now 40 years old engine dividing machines were scrapped, and the latest DIWA multiple-line dividing tooling equipment was employed making modern precision dividing stamps for reduction scales. The DIWA dividing machines were scrapped in 1992, as the stamps could be made in a combination of photo-etching and CNC machining.
The DIWA factory and the shops were torn down and a supermarket was built instead.
In 1980 Mr. Larsen sold UTO to Flemming Holme. He moved the factory to Vejle and later to Vonge, and continued making custom-developed slide rules, and very special slides, beside a range of drawing
articles. In the later years, when the competition grew, UTO survived by making artillery slide rules for the Danish Ordnance and the German Bundeswehr. When the production runs grew smaller, UTO, being a formerly specialised factory of 70 employees, had plenty of machinery standing ready to run the different shapes, without the need to change the set-ups, thereby remaining competitive.

John Kvint joined Linex in 1967 and soon after got permission from the Linex management to act as spare-time scale developer for CRETEX. In 1993 Linex was sold to Bantex, an office-supply firm and John got a part-time employment at Linex and after working as consultant he established production in 1995 under the name John Kvint Plastteknik.
He is subcontracting specialities like long splines and string-guided parallel rulers for Linex. He also subcontracted some of UTO's products, and after Flemming Holme decided to close UTO in 2002, John continues some of the UTO products, such as the double-slide slide rule for X-ray exposure, that will be the subject of one of the other IM2003 sessions.


## Design and Printing of Scales

## John Kvint

Various technical methods have been used during the last decades of slide rule manufacture. As manufacturing methods improved - so did the designing of scales change.

1) History of scale appearance
2) Drawing a logarithmic scale
3) Making scales by single line Cutting/engraving
a) Manual methods
b) Semi-mechanical methods
c) Mechanical methods
d) Colour-coding of scales
4) Designing multiple line impression stamps.
a) Etched brass stamps
b) Etched steel stamps
c) Ground steel stamps
d) Segmented steel stamps
e) Brass body with steel knives.
5) Direct processes without stamps
a) Silk-screen printing of scales
b) Tampon-printing of scales
c) Laser-engraving of scales

## History of scale appearance - the "railroad track" design

From very old times it was common practice to add a scale to every technical drawing, mainly architecture drawings of buildings, as there was no standardisation of scale reductions, and both architects and machine builders were accustomed to having an individual scale on drawings. (1)

Every draftsman was taught how to deal with the individual scale. One simply decided what reduction to use to fit the building unto whatever goatskin was used, and started the drawing by constructing the actual scale in a bottom corner of the drawing.
As all measurements thereafter were transferred to the drawing using a pointed compass, the draftsman could finish the drawing with very few instruments, such as compass, silver(-pencil) (2) and a ruler (straightedge).
Sometimes templates for duplicating curves or other odd shapes had to be made additionally.
The scale was drawn in a practical length = what could be "taken off" by the compass. Making a set of at least three or more parallel lines, the compass was used to point out a set of feet or fathoms. Next step was using the compass for subdividing into halves, then into quarters, and so on until the smallest subdivision needed.


Figure 1: A scale was constructed on every technical drawing

The division lines were drawn between the compass intersections. When considered accurate, one traced the division lines using ink and as guidance to obtain equal lengths one simply started and finished the division lines on the proper parallel. When the ink was dry, one normally erased the construction and only the inked railroad track scale remained.


Figure 2: Using a compass to intersect to subdivisions

## Railroad track on tools

Making permanent feet- or yard-scales of stable materials such as wood, brass, silver or ivory, the "toolmaker" (3) used the just mentioned dividing technique, but instead of tracing the division lines with ink, he had to engrave the lines using a needle. And because he had to use a certain force to engrave the hard material, he had to rely on the parallel lines for starting and ending his forceful action.
Even as dividing machines appeared (4) first as company secrets, later became more common, the railroad track design remained. It is still in use, mainly as scales on nautical articles (5) and charts.

## The modern scale

When machines took over the dividing, they could engrave the length of the division lines accurately, and there was no real need for the parallel track lines, except for obtaining the general appearance of a true scale. People also were used how to read the scales and how to utilize the track shaped rectangles to interpolate subdivisions (6). But the metric and decimal system took over and it was simply easier to read the division lines only having 5 and 10 emphasised and with only 4 short lines in between.
The logarithmic scale having uneven distance between division lines, presented a problem, and people had to learn how to read the scales correctly (7)


Figure 3: Different reading techniques to be applied

## Drawing a logarithmic scale

To keep things simple and allowing comparing the methods, we choose the normal D-scale of a slide rule.
We decide the length of the decade to be 250 mm .

We decide the lines to be reading:
1 to 2 by increments of $0.01=100$ divisions
2 to 4 by increments of $0.02=100$ divisions 4 to 5 by increments of $0.05=20$ divisions 5 to 10 by increments of $0.05=100$ divisions adding a line 1 we have a total of 321 division lines.

Picture of a slide rule as above indicated


Figure 4: An A-scale having 321 division lines

Taking a logarithmic table, we multiply the mantissa of the division lines with 250 and write a new table:

| division line | mantissa | scale position |
| :--- | :--- | :---: |
| $\quad 1$ | 0 | 0 |
| 1.01 | 0.00432 | 1.08 mm |
| 1.02 | 0.00860 | 2.15 mm |
| 1.03 | 0.01284 | 3.21 mm |
| 1.04 | 0.01703 | 4.26 mm |
| $\ldots$. | $\ldots .$. | $\ldots .$. |
| and so on to |  |  |
| 2 | 0.30103 | 75.26 mm |
| $\ldots \ldots .$. | $\ldots \ldots \ldots$. | $\ldots \ldots \ldots$ |

The segment 2 to 4 is precisely like 1 to 2 .
Therefore we just add 75.26 mm to the first table:

| 2.02 | $1.08+75.26=$ | $76,34 \mathrm{~mm}$ |
| :--- | ---: | :--- |
| 2.04 | $2.15+75.26=$ | 77.41 mm |
| $\ldots \ldots .$. |  |  |
| til 4 | 0.60206 | 150.52 mm |


| From 4 to 5 we again look in the logarithmic table: |  |  |
| :---: | :---: | :---: |
| 4.05 | 0.60746 | 151.87 mm |
| 4.1 | 0.61278 | 153.20 mm |
| 4.15 | 0.61805 | 154.51 mm |
| $\ldots \ldots$. | $\ldots . . .$. | $\ldots . . . .$. |
| until 5 | 0.69897 | 174.74 mm |


| From 5 to 10 we just add the first table again, giving: |  |
| :---: | :--- |
| 5.05 | $174.74+1.08 \mathrm{~mm}=$ |
| 5.1 | $174.74+2.15 \mathrm{~mm}=$ |
| $=$ | 176.89 mm |
| $\ldots \ldots$. | $\ldots \ldots .$. |
| until 10 | 1.0000 |

Now we have a table for all the lines we need for a D-scale. And also for the C-scale on the slide!
If you want to draw this scale, nobody can hand-draw a line with the accuracy of 0.01 mm .
Going back to the table above, multiply by 4 and make an oversize scale:

| division line | mantissa | true scale position | oversize (x4) |
| :---: | :--- | :--- | :--- |
| 1 | 0 | 0 | 0 |
| 1.01 | 0.00432 | 1.08 mm | 4.32 mm |
| 1.02 | 0.00860 | 2.15 mm | 8.60 mm |
| _..... | $\ldots . . .$. | $\ldots .$. | $\ldots$ |
| and so on.......... 361 times ??? |  |  |  |

No! Be smart!
Only make the oversize master-scale for 1 to 2 and the filling section 4 to 5 .
The missing oversize designs being 2 to 4 and 5 to 10 were made just by a relocation of the first master scale.
From each true scale position marks draw a division line.
We save this drawing of a scale for later purposes.

## Making scales by single-line cutting/engraving

## a) Manual methods

Making the lines one by one, you have no production. Even a very simple slide rule having only C and D scale, will require 642 lines hand-positioned and cut into the white ivory or celluloid.
b) Semi-mechanical methods

A lot of different dividing machines have been developed in firms making scales and rulers. The 1863 catalogue from Keuffel \& Esser mentions precision scales as being "machine divided".
Some machines must have been re-invented if not just copied from earlier patent descriptions.
One example is the UTO dividing machine developed 1941 by "Kruuse and Larsen Instrumentfabrik" and used with few improvements added, until 1993.


Figure 5: Larsen divider - one operator cranked the spindle, the other cut the divisions

It looked like a lathe. There is a horizontal slide-way with a moving slide. And parallel to that there is a fixed clamping device to hold the material to be engraved. Centrally in the slide-way is located a long spindle approximately 25 mm thick and cut with a thread assumed to be 1 " BSW having a pitch of 3.175 mm per revolution. A loose nut connected the spindle and the moving slide. To take up any play in the nut, there was made an axial preload by means of a wire and a pulley in the end of the apparatus. In the wire was hung a weight.

To the left where a lathe has the clamp, there was a large circular disc - approx. 600 mm diameter mounted on the spindle. An operator turned the spindle using a handle on the disc, thereby making the slide move.
The disc had numbered radial markings like a protractor. Moving to the next number resulted in moving the slide $0,01 \mathrm{~mm}$. A counter was fitted to count the revolutions of the disc.
A second operator moved a little knife fitted on the moving slide. The knife could only move perpendicular to the movement of the slide, and only very short strokes.
A table was made indicating the number of revolutions and the number of marks to turn the spindle to achieve the proper position of the divisions to be cut.

The first operator turned the disc according to the table and kept the handle still, while the second operator moved the knife to cut the line. This second operator had a drawing close to the knife to tell when the lines should be short, and when to be long.

Quite simple - but - there were many errors. And the waste was near 50 percent.
c) Mechanical methods


Figure 6: Duelunds divider had exchangeable spike-drums and exchangeable clamping tables
The Linex dividing machine from 1938:
Using the threaded spindle and adding a rotary drum of 159 mm in diameter, you have a circumference of 500 mm per revolution. If the spindle has a pitch of 5 mm , the movement of 250 mm requires 50 revolutions. We multiply 50 with the circumference of the drum, and get a spiral length of 25000 mm .
This will allow to position in one hundredths of a millimeter.
Using simple skill, one punches and drills a hole for every line position, and fit a short steel pin in every hole.
Now we have a "Swiss music box-machine" (playing a decade!).

At the other end of the mechanical dividing machine, we have a table moving lengthwise with the drum $=250 \mathrm{~mm}$. An electric switch stops the drum rotation, and activates a system of knives.

A slide was fitted above the table and moving perpendicular to the table over a preset distance, like 5 mm . On the slide was fitted a knife controlled by a fixed cam. The transverse movement will allow the knife to drop down - make a cut and then rise again.

The drum pins had three lengths, operating a switch with 3 levels that controlled the line length.
To increase production, the table was fitted with 6 rows of workpiece-holders and 4 slides with 6 knives on each. Thereby 24 scales were cut simultaneously. And the only manual job was clamping the 24 bodies correctly.

The pinned drums was exchangeable. When an appropriate amount of slide rules have been cut with one scale, the next drum, say square-division - being two decades - was cut, and after that maybe the cubic scale having three decades.

## d) Colour coding of scales.

For production the black lines were cut first. Then the numbers were punched or pressed.
Then lines and numbers were filled with black lacquer and dried.
Excess colour was wiped off when still wet, and later, when dry, the surface was sanded to be almost smooth.
Next the red lines were cut and the red numbers were pressed.
Then the red scales were filled with red lacquer, wiped off and dried.
The red colour would not show on the black divisions.
Now the material protruding from cutting and punching numbers was sanded to make a smooth and even surface. As the lines and numbers were deep into the white material, the slide rule could withstand this processing.

## Designing multiple-line impression stamps

It soon became obvious that cutting line by line was not ideally productive. A lot of thought was put into more productive methods.

## a) Etched brass stamps

Designing the decades needed for a particular slide rule, could be done using manual or more sophisticated methods. A drawing was made 4 times oversize, to exclude manual errors, and then reduced to normal size in a reproduction camera.
After reproduction you prepare two sets of film, one film for black lines and numbers, and the other for red lines and numbers. From these films a reprographic shop makes a set of etched brass cliches.
Using a press, all the divisions of the same colour could be made in one short stroke of the press.
As mentioned before, the black part was made first - then the red part.
A cheap method was the hot-foil impression. A black impression foil was placed under the brass stamp and the black lacquer from the foil was deposited in the impressions made in the surface of the slide rule.
An example is the Aristo 0903 Scolar. See figure 7.
Brass stamps can be used up to 10.000 times when cold and 20.000 times when used in the hot process.

## b) Etched steel stamps

In a similar manner as for the brass stamp, steel could be etched. To overcome the time needed for a steel etching, usually the waste between lines and indications was machined away.
Steel stamps can be hardened after routing or using tungsten cutters for relief, even be made of pre-hardened steel, and will then withstand 50.000 to 100.000 impressions.

## c) Ground steel stamps

For a shorter period in time, approximately from 1970 to 1980, computer controlled machinery being more common, machine control allowed the division line stamps to be ground.


Figure 7: Aristo 0903 is a typical hot-stamp imprinted slide rule


Figure 8: The arrows show the two directions of grinding
The grinding stone was shaped in profile like a sharp knife, and moving the grinding wheel sidewise along a hardened steel bar, one could grind away the material between the lines.
to make the different length of the lines, a similar grinding operation was performed on one side of the steel bar. Many steel bars were assembled to one block stamp, some bars being only the numbers.

## d) Segmented steel stamps

Making a tool with multiple knives, you die-punch the needed number of knives, all having a practical outer shape, and a working length of say 2-3 and 4 mm .
Steel spacers, being of a similar shape, but with no cutting means, were ground to accurate thickness to space the knives according to the divisions to be made.


Figure 9: Spacers and three knifes, size approximately 25 by 20 mm

Knives and spacers were then mounted onto a bar, and a set of clamping bolts kept the unit together.
The unit - carefully used - was capable of pressing 5-10.000 scales and could make another 10.000 after a regrinding.
The segmented stamp was impractical for slide rules, but was used for 10 years making reduction scales.
e) Brass Body with inserted steel knives.

A brass bar say $4 \times 40 \mathrm{~mm}$ was divided using a modified single-line cutting machine like the Kruuse \& Larsen single-line divider in use at DIWA.
Instead of cutting a line across a 5 mm edge of the brass bar, a small electric circular saw was mounted.
The saw made a clean cut 0.1 mm wide and 2 mm deep.
From a hardened sheet of steel, small leaves of 4 by 3 mm were die-punched. Along the 4 mm side they were cut in three different shapes.
These tiny leaves were mounted in the saw-cuts in the brass bar, according to the design to make a readable scale. The bar with lines could be used singly for one scale - or be mounted with more bars to allow a full faced impression. The numbers were placed in brass bars between the line division bars, or in cut slots in the line-bar.


Figure 10: Die-punch numbers fitted into grooves, cut in the knife-holder bar

## Direct processes without stamps

## a) The photo-etching process.

For the photo-etching process you prepare an artwork similar to the artwork used making a brass stamp, as described above. But usually you multiply the images to make four or five slide rules in one operation.
White PVC plates are cut to fit the area for 5 slide rules - approx. $300 \times 300 \mathrm{~mm}$.
The plates are placed in a light-tight horizontal spinner. Here a special photographic emulsion called a resist is poured on the spinning plate to make an even layer, and after that the plates are dried when still remaining in the spinner. The plates are taken out, exposed to contact with the artwork, whereby the resist hardens. The unexposed resist is washed away, and black solvent-colour is wiped on. The process is repeated for the red colour. Finally the remaining resist is dissolved and washed away. The multi-image plates are next cut apart to make bodies or slides.

## b) The silk-screen process

Again one film for each colour is needed. Again you print 4 or 5 slide rules per sheet of PVC.
The films are copied onto a (silk)-screen (actually it is made of polyester). In the silk screen process the colour is squeezed through the screen onto the PVC sheets.
After the paint is dry, the next colour can be applied.
The silk-screen method is widely used for making slide-charts.

## c) The tampon-printing process

The pattern to be printed is etched into a plate of steel or plastic. Then an amount of ink is wiped on the etched plate and the plate is scraped clean leaving only ink in the etchings.
Next a silicone tampon is pressed on the pattern and lifted up. As the ink is partially dry on the surface, it becomes sticky and adheres to the tampon. Then the tampon is pressed onto the subject which is our slide rule. The ink on the tampon has now dried a little on its surface - whereby it becomes sticky and adheres to the slide rule. You repeat the process for the next colour.


Figure 11: A tampon oscillates between an etched master $\mathbf{E}$ and the slide rule $\mathbf{S R}$

## d) Laser engraving

A laser makes concentrated beams of light with such high energy that it can penetrate steel.
Using low energy one can manage to make just an engraving, but as the machines are very precise, you usually get precision in the range of 0.001 mm . As the light has to wipe over the full scale the process is considered too costly for scale making, and at UTO we sometimes use the method for cursor lines.


Figure 12: A laser beam causes the material to evaporate, to make grooves for the colour
The engraving is filled with black and/or red ink.
There is no film made. A PC program controls the lens movement of the laser.

## Ready Reckoners and Tabular Calculators

## Bruce O.B. Williams

## Demanding calculations

Commercial calculations before the advent of electronic devices were very demanding, particularly when in nondecimal unit such as:

- 2 yard $2 \mathrm{ft} 61 / 2$ inches @ $19 \mathrm{~s} 113 / 4 \mathrm{~d}$ per yard?.
- 7 tons 5 cwt 3 qrs 17 lbs @ 41.8/- 4d per ton?
- the interest due on $£ 127$ for 137 days @ $4.75 \%$ p a , in $£$ s d
- The area of an object 12 yard 2 ft 6 in by 1 ft 7 in .

For accurate calculations of this sort, slide rules were not accurate enough. Machines were too expensive. Logarithms were too complicated and slow.

The need was met by Ready Reckoners and Tabular Calculators, using pre-calculated results.


Figure 1: Devices available for Multiplication

## Ready Reckoners

Ready Reckoners were produced by the million: Figure 2 shows the number of new titles each decade listed in the British Library catalogue.


Figure 3：A Ready Reckoner page ${ }^{\text {ii }}$

Ready Reckoners came in many varieties，but they were not all that easy to use．You often had to do a calculation in several＂partial products＂ and then add these yourself．For example，if you want
7 tons 5 cwt 3 qrs 17 lbs＠41．8／－4d per ton， first work out the 7 tons，then the 5 cwt ，then the 3 qrs，then the 17 lbs ，each probably from a different page of the book

So people looked for ways to make life easier， and came up with the idea of somehow mechanising the tables．These were the Tabular calculators，a term that seem to have been widely used both in GB and in the USA， particularly in the Patents．

## Tabular Calculators

Tabular calculators are Ready Reckoner pages cut up and pasted on to Boards，Discs， Cylinders and Rolls．

Free－standing Tabulators are little known But those attached to measuring devices， particularly scales for retail sales，were made in millions．This paper mainly covers Free standing devices，but concludes with a note on Computing Scales．

For free－standing devices I have found over 300 Patents；but only about 40 surviving examples． These are in museums and private collections，

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| $2{ }_{4}^{4}$ | －－ 1 | 55 |  | 105 | － 13 | 155 | － $2 \frac{1}{2}$ | 224 | － | － $3 \frac{1}{2}$ | 1760 |  | 2 31 |
| $5 \frac{1}{2}$ | －－ 1 | 56 | 1 | 106 | － 13 | 156＊ | － $2 \frac{1}{2}$ | 225 | － | －3 ${ }^{\frac{3}{4}}$ | 1800 | － 2 | 241 |
| $6 \frac{1}{2}$ | －－${ }^{\frac{1}{4}}$ | 57 | 1 | 107 | － $1 \frac{3}{4}$ | 157 | － 212 | 230 | － | －3 ${ }^{\text {a }}$ | 1900 |  | 25 |
| 8 | －－$\frac{1}{4}$ | 58 | 1 | 108＊ | ＊－13 | 158 | － $2 \frac{1}{2}$ | 235 | － | －3 ${ }^{\text {a }}$ | 2000 | － 2 | 271 |
| 9 | －－ 1 | 59 | 1 | 109 | － $1 \frac{3}{4}$ | 159 | － $2 \frac{1}{2}$ | 240 | － | $-3 \frac{3}{4}$ | 2240 |  | 211 |
| 10 | －－ | 60＊ | 1 | 110 | － $1 \frac{3}{4}$ | 160 | － $2 \frac{1}{2}$ | 250 | － | $-4$ | 2500 |  | $3 \quad 31$ |
| $11$ | －－$-\frac{1}{4}$ | 61 | 1 | 111 | － 13 | 161 | － $2 \frac{3}{4}$ | 256 | － | － 4 | 3000 |  | 311 |
| 12＊ | －－${ }^{4}$ | 62 | 1 | 112 | － 1 | 162 | － $2 \frac{3}{4}$ | 275 | － | $-4 \frac{1}{2}$ | 4000 | － 5 | 5 21 |
| 13 | $-\quad-1$ | 63 | 1 | 113 | － 2 | 163 | － $2{ }^{3}$ | 280 | －－ | － $4 \frac{1}{2}$ | 5000 |  | 661 |
| 14 | －－ 1 | 64 | 1 | 114 | 2 | 164 | －23 | 300 | － | －43 | 5500 |  | 72 |
| 15 | －－$\frac{1}{4}$ | 65 | －13 | 115 | 2 | 165 | － 23 | 350 | － | －51 $\frac{1}{2}$ | 6000 |  | 7 93 |
| 16 | －－ | 66 | － 11 | 116 | 2 | 166 | － 23 | 365 | －－ | －53 | 6500 | － 8 | 8 5 ${ }^{\frac{3}{4}}$ |
| 17 | －－$\frac{1}{2}$ | 67 | － $1 \frac{1}{4}$ | 117 | 2 | 167 | － 23 | 400 | － | － $6 \frac{1}{4}$ | 7000 | － 9 | 9 12 |
| 18 | －－${ }^{1}$ | 68 | － 14 | 118 | － 2 | 168＊ | $-2 \frac{3}{4}$ | 450 | － | － $7 \frac{1}{4}$ | 7500 | － | 991 |
| 19 | －－$\frac{1}{2}$ | 69 | $-14$ | 119 | － 2 | 169 | $-2 \frac{3}{4}$ | 480 | － | － $7 \frac{1}{2}$ | 9000 | － 11 | 118 8 |
| 20 | －－$\frac{1}{2}$ | 70 | － $1 \frac{1}{4}$ | 120＊ | ＊－2 | 170 | － $23 \frac{3}{4}$ | 500 | － | $-8$ | 10000 | － 13 | $13-\frac{1}{4}$ |
| 21 | －－$\frac{1}{2}$ | 71 | － $1 \frac{1}{4}$ | 121 | 2 | 171 | － 23 | 516 | － | －81 | 11000 | 14 | 14 |
| 22 | －－$-\frac{1}{2}$ | 72＊ | － 11 | 122 | 2 | 172 | － 23 | 520 | － | －81 | 12000 | － 15 | 15 71 |
| 23 | －－$\frac{1}{2}$ | 73 | － 11 | 123 | 2 | 173 | － 23 | 540 | － | $-8 \frac{1}{2}$ | 13000 | 16 | $1611 \frac{1}{4}$ |
| $24 *$ | －${ }^{\frac{1}{2}}$ | 74 | － 14 | 124 | 2 | 174 | － 23 | 550 | － | －83 ${ }^{\frac{3}{1}}$ | 15000 | － 19 | 19 6 $\frac{1}{2}$ |
| 25 | － 1 | 75 | － 14 | 125 | 2 | 175 | － 23 | 560 | － | －83 ${ }^{\frac{3}{4}}$ | 17000 |  | 2 1委 |
| 26 | －－$\frac{1}{2}$ | 76 | － 11 | 126 | 2 | 176 | － 23 | 564 |  | － 9 | 19000 |  | 49 |
| 27 | －－$\frac{1}{2}$ | 77 | $-14$ | 127 | 2 | 177 | $-3$ | 570 | － | $-9$ | 20000 | 1 | $6-\frac{1}{2}$ |
| 28 | －－ 1 | 78 | － 11 | 128 | － 2 | 178 | － 3 | 580 | － | － 91 | 25000 | 112 | 12 63 |
| 29 | －－$\frac{1}{2}$ | 79 | － $1 \frac{1}{4}$ | 129 | － 21 | 179 | $-3$ | 590 | － | － 91 | 30000 | 119 | 19 －3 |
| 30 | - － 1 | 80 | － $1 \frac{1}{4}$ | 130 | － 21 | 180＊ | $-3$ | 600 | － | －912 | GROSS． |  |  |
| 31 | －－$\frac{1}{2}$ | 81 | － $1 \frac{1}{2}$ | 131 | － 21 | 181 | － 3 | 625 | － | $-10$ |  |  |  |
| 32 | －－$\frac{1}{2}$ | 82 | － $1 \frac{1}{2}$ | 132＊ | － 21 | 182 |  | 640 | － | $-10$ | 2 |  | － $4 \frac{1}{2}$ |
| 33 | －－${ }^{4}$ | 83 | － $1 \frac{1}{2}$ | 133 | － 21 | 183 | － 3 | 650 | － | $-10 \frac{1}{4}$ | 3 |  | －6\％ |
| 34 | －－${ }^{4}$ | 84＊ | －11 | 134 | － 21 | 184 | － 3 | 700 | － | $-11$ | 4 |  | － 9 |
| 35 | －－${ }^{4}$ | 85 | －1 $1 \frac{1}{2}$ | 135 | － $2 \frac{1}{4}$ | 185 | － 3 | 750 | － | －11委 | 5 |  | $-11 \frac{1}{4}$ |
| 36＊ | －－${ }^{\frac{3}{4}}$ | 86 | －13 | 136 | － 21 | 186 | － 3 | 800 | － | $1-\frac{1}{2}$ | 6 |  | $1 \frac{1}{2}$ |
| 37 | －－$-\frac{8}{4}$ | 87 | － $1 \frac{1}{2}$ | 137 | － 21 | 187 | 3 | 850 | － | $1{ }^{1} \frac{1}{2}$ | 7 |  | $3{ }^{3}$ |
| 38 | －－$\frac{3}{4}$ | 88 | －13 | 138 | － 23 | 188 | 3 | 900 | －－ | 123 | 8 | － | 6 |
| 39 | －－$-\frac{3}{4}$ | 89 | － $1 \frac{1}{2}$ | 139 | － $2 \frac{1}{4}$ | 189 | 3 | 950 | － | 13 | 9 |  | 188 |
| 40 | －－${ }^{3}$ | 90 | － $1 \frac{1}{2}$ | 140 | － 21 | 190 | 3 | 1000 | － | 133 | 10 |  | $110 \frac{1}{2}$ |
| 41 | －－$\frac{3}{4}$ | 91 | － $1 \frac{1}{2}$ | 141 | － $2 \frac{1}{4}$ | 191 | － 3 | 1016 | － | 14 | 11 |  | 2 －量 |
| 42 | －－${ }^{\text {a }}$ | 92 | －17 | 142 | － $2 \frac{1}{4}$ | 192 | － 3 | 1094 | － | $15 \frac{1}{4}$ | 12 | － 2 | 3 |
| 43 | －－${ }^{3}$ | 93 | － $1 \frac{1}{2}$ | 143 | － 21 | 193 | － 34 | 1100 | － | $15 \frac{1}{4}$ | 13 |  | $2{ }^{51}$ |
| 44 | －${ }^{4}$ | 94 | － $1 \frac{1}{2}$ | 144＊ | － 21 | 194 | － $3 \frac{1}{4}$ | 1120 | － | 1 51 | 14 |  | $27 \frac{1}{2}$ |
| 45 | －－${ }^{4}$ | 95 | － $1 \frac{1}{2}$ | 145 | － $2 \frac{1}{2}$ | 195 | －31 | 1200 | － | $16 \frac{3}{4}$ | 15 |  | 293 |
| 46 | －－$\frac{3}{4}$ | 96＊ | － $1 \frac{1}{2}$ | 146 | － $2 \frac{1}{2}$ | 196 | －34 | 1250 | － | 1783 | 16 | ， |  |
| 47 | －－3 | 97 | － $1 \frac{3}{4}$ | 147 | $-2 \frac{1}{2}$ | 197 | －31 | 1300 | － | $18 \frac{1}{2}$ | 17 |  | 324 |
| 48＊ | －－3 | 98 | － $1{ }^{\text {星 }}$ | 148 | － $2 \frac{1}{2}$ | 198 | － $3 \frac{1}{4}$ | 1350 | － | 194 | 18 |  | 3 4 ${ }^{2}$ |
| 49 | 1 | 99 | －13 | 149 | － $2 \frac{1}{2}$ | 199 | － $3 \frac{1}{4}$ | 1400 | － | 110 | 19 | － 3 | $36 \frac{3}{4}$ |
| 50 | 1 | 1100 | －13 | 150 | － $2 \frac{1}{2}$ | 200 | －31 | 1500 | － | $111 \frac{1}{2}$ | 20 | 3 | 39 | and some in contemporary journals，descriptions，advertisements，Instruction manuals and the like



Figure 4：Patents per decade mainly GB and USA ${ }^{\text {iii }}$

## Examples of the main categories

I describe a few of the 40 actual devices that I have identified, and some examples of patents. In a future paper I hope to classify the actual, and the patents, by application, method of construction and country of inventor.

## Leavitt, USA, Disc $1845^{\text {iv }}$

This is an interest calculator, but for $6 \%$ only. $6 \%$ was the legal maximum in US at the time. It has 19 circles and 57 sectors, giving 1083 results. Made in USA.

## Van der Weer

This is the earliest cylinder example I have found. A USA Patent
The idea was that school children would build their own calculator by making their own table and gluing it on a cylinder.


Figure 5: Van der Weer, USA, Cylinder Patent 1846

Figure 6: Baranowski,
Russia, Slides for


Wages $1847^{\text {v }}$

## Baranowski

Baranowski was a Polish hustler He sold 100 of these devices to a Russian prince who owned gold, copper and malachite mines. The machine is for calculating wages. There are examples in C.N.A.M., France and the Science Museum.

## Peale



Peale's caiculator.

Figure 7: Peale, USA, cylinder 1865

American Wages machine described in an article in Scientific American 1865. It cost \$ 6-8-10 according to size and seems to have been properly marketed .

To find the amount of wages necessary to be paid for $93 / 4$ days at the rate of $\$ 1275$ per week or $\$ 2121 / 2$ per day. Turn the cylinder by means of the milled heads at the left until the figures $93 / 4$ on the left-hand column, appear to view; then above the figures $\$ 1275$, denoting the rate of wages, on the outside of the case, will be found 20.72 -which Is the amount to be paid.

## Maurand



These three examples will suffice to shew the manner of operating with this. instrument, it being understood that when it is desired to operate the dial plate or tablet on which are the measures to be converted or the calculations tobe made, as when interestat a certain rate is to be ascertained, is tobe first placed beneath the transparent lid ".

## Chambon

This is an interest calculator by Chambon. He made some other similar devices for children. This in the Science Museum and the Cyber museum

Turn the top cylinder for thousands, the second for hundreds, the third for tens and the lower for units.

Figure 9: Chambon, France Cylinders 1880 Courtesy Science Museum


## The Great Machines

## Meilicke

 Science Museum

We come now to the "GREAT MACHINES": Meilicke, Hines and Roberson. These three took out between them a dozen patents from 1904 to 1915.

The patent of the Meilicke explains how it works for calculating the interest on notes.
"The card upon the drum is divided into three hundred and sixty-five divisions corresponding to the days of the year, and at the right of the card and at its edge which lies adjacent to the calendar figures are placed running from " 0 " to . "365 " in succession, so as to constitute the day-strip. Thus we have two bands upon which may be indicated the months and dates of the months of the notes, while we have a strip which may indicate the number of days for which the note is to run."
The Meilicke is an interest calculator. This is the outside of the Science Museum machine. It has a light inside to
 illuminate the drum Other versions had a hand lens attached This model was built about 1915.

Figure 11: Meilicke
interior Courtesy
Science Museum

The Meilicke with the lid off. This is no lap top. It is made of cast iron.
There are examples in private collections and some have been offered on eBay.

## Hines



Figure 12: Hines Scotland 1907 Cylinders Courtesy Science Museum

Figure 13: Hines Instructions

## HINES' WAGE AND COSTING CALCULATOR

(PATENTED)


SIZE 24 in. $\times 5$ in. $\times 4 \frac{1}{2}$ in.
Saves Time and Money. Only One Calculation in View.
Mistakes are Impossible.
No Mental Additions of $\frac{1}{4}, \frac{1}{2}$ and $\frac{3}{4}$ Hour Amounts.
No Turning over Pages.
The Figures are Large and Easily Read.
No Mass of Figures to Confuse the Eye and Lead to Errors in Selecting the Right Calculation.

Robertson


Figure 14: Robertson Scotland Rolls 1910 Exterior, Courtesy Science Museum


Figure 15: Robertson Rolls 1910 Interior, Courtesy Science Museum

The magnificent Robertson machine. Made for use by the maker in Glasgow. It was marketed feebly from 1910.

It does not look a trivial task to change the roll! For a full description see the Tercentenary Hand book ${ }^{\text {vii }}$ from which" we read: "The present model, as illustrated, is set upon a desk-table. It has four distinct faces, each face showing different sets of equivalents. The operator, by simply pressing a small key, brings the required
face opposite him, with the controlling handles ready for use.
Each face of the machine with its printed records may be likened to a book with 200 or 300 pages open at the one time, allowing the machine to be operated, while showing the full sets of equivalents. The operator is thus enabled in many instances to do some thirty different calculations in five minutes, without requiring to re-set the machine.
To the sloping desk in front of the machine is fitted a further series of calculated records of equivalents, in order to enable the operator, having found an answer in the main machine, to convert it into other equivalent values."

## Two devices combining: log scales with cylinders

These are not tabular but use a cylinder to extent the range of a slide rule.

## Farmar's Profit Calculating Rule .



Figure 16: Farmar UK 1912 Cylinder + Slide Rule From a private collector

From GB Patent 12717 May 30 1912:
"A slide rule, according to my invention, essentially comprises a flat slide and a revoluble roller (or rollers), tube, cylinder, or the like, fitted or mounted in the rule, and arranged to operate with said flat slide, on one of which, preferably the roller, are arranged a plurality of logarithmic graduations of money expressions, and on the. other of which are arranged logarithmic graduations of discount, profit on turnover, or profit on cost, or any two or all of such graduations."

## Arnolds's Mortgage Computer

This is described in US Patent 3298604 of 1967.
Picture from a private collector ${ }^{\text {viii }}$.

Figure 17: a Arnolds instruction leaflet

Devices still available for collectors.
Recently acquired by the author. Two Russian Tables with Cursor ${ }^{\text {ix }}$


$$
\begin{aligned}
& \text { for instant answers to your mortgage } \\
& \text { computation problems... } \\
& \text { THE ARNOLD MORTGAGE COMPUTER }
\end{aligned}
$$

Is easy to use. Convenient. Can be used in office, home or car ... whenever or wherever it's needed! measures only romes money-back basis. See how it can.
SOLVE PROBLEMS LIKE THESE IN SECONDS

1. What is balance due on a $\$ 9430.00$ loan with $7 \%$ annual interest after thirty-six

2 How many months does it
2. How many months does it take to reduce a $\$ 5300$ principal to $\$ 3920$ with $7 \% \%$
3. How long does it take to amortize a $\$ 6200$ loan at $61 / 2 \%$ with $\$ 150$ monthly
payments?
4. What principal amount will a $\$ 12.50$ monthly payment, with interest at $8 \% \%$,
amor mears?

BUILT FOR YEARS OF DAY-IN, DAY-OUT USE
The Arnold Mortgage Computer has a sturdy, gold anodized aluminum case. Front scale is made of shatter resistant Plexiglas ${ }^{*}$. All interest markings are indelibly screened on tough Mylar ${ }^{8}$ film,
The Arnold Mortgage Computer is easier to use than interest tables. Try it yourself, on a
10 day, money 10 day, money-back guarantee basis.


Sanks - Savings \& Loans - Architect P. BOX 1203 PUNTA GORDA, FLORIDA 33950
Accountants - Realtors - Insurance Companies - Real Estate Management Firms

Figure 18: Russian tables

## 

| 11 |  | $13$ |  | 15 | $16$ | $17$ |  |  |  |  |  |  |  | 25 |  |  |  | $29$ | $3132$ | $33$ | $\sqrt[34]{34} \sqrt{35}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 010 | 0.012 | 0.014 | 0.016 | 0.019 | 0.02 | 0.024 | 24020 |  | 0.033 | 0.03 | 0.040 | 90.04 | 0.0 | 0.05 | 0.0 | 0.062 | 0.0870 | 0.0720 | B082 | 970.09 | 0,10 | 00.11 |  | 20,130 |  |  |
| , |  | ,030 | .03 | 0.039 | 90.0 | 0.050 | 0.05 | -06330 | oof | 0.07 | 0.088 | a | . 10 | 0, 113 | 012 | 0,13 | 01440, | 0,159016 | 01770, | 0,20 | 0,210 | 0,2 | ,250,26 | 0,270, |  |  |
|  | 20.038 | . 094 | 0.05 | 20.060 | 00.068 | 0.078 | 0.088 | 60.0960 | 0.107 | 0.18 | 0.130 | 0.143 | 157 | 0.170 | 188 | 0,20 |  |  |  |  | 0,310,3 |  | , 370.38 | 0 0,410 |  | 250,4 |
| OOA5 | 50.053 | 2062 | 0.073 | 0.084 | 0.095 | 0.10 | . 120 | b0.133 | 0.147 | 0.163 | 0.18 | 10.195 | 0.21 | 0.23 | 0.25 | 0,27 |  | $0.310,33$ | ${ }^{0.360 .38}$ |  | 0,9330,9 | 50,48 |  |  |  |  |
| 0.062 | 20.073 | .085 | 0,097 | 0.10 | 0.129 | 0.190 | 0.156 | f.iza | 0.190 | 0,21 | 0,23 | 0,25 | 0.27 | 0,29 | 0.32 | 0.39 | 0,37 | 0.390 .48 | 450,48 |  | 0,590,5 | \%0,00 |  | 7 O 290 |  |  |
| 80 | 00.093 | 0.108 | 0.123 | 0.140 | 0,155 | 0.75 | 0,194 | 0,21 | 0.23 | 0.26 | 0,28 | 0,31 | 0.33 | ${ }^{0.36}$ | 0.39 | 4 |  | 0.480 | 55.059 |  | 0,660, |  |  |  |  |  |
| 98 |  | 0.132 | 0.15 | 169 | 0,189 | 0,21 | 0,23 |  | 0,28 | 0,31 | 034 | 0,37 | 0.40 | 0 | 0.48 | 0.30 |  | 0.580 .61 | 0.660 .70 |  | 0,780, 8 | 30,880 | 0830,97 |  |  |  |
| 0.120 | 0,138 | 0.58 | 0.179 | 0,20 | 0,22 | 0.25 | 0,28 | 0.30 0 | 0.33 | 0.36 | 0.40 | 0.43 | 97 | 0.50 | 0,54 | 0.58 | 0.630. | 0,670,72 | 0,770,02 |  | 0,920,9 |  |  | 1,201,2 |  |  |
|  |  |  |  |  |  |  |  | 0,36 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

These are very simple tables with a cursor to guide the eye. One is for forestry calculations and the other for holidays.

## Telegraph Calculator Cylinder ${ }^{\text {x }}$



## Figure 19: The telegraph Calculator Cylinder

This was for calculating the cost of a telegram to different destinations and for varying numbers of words.

## Computing Scales ${ }^{\text {xi }}$

They are to price goods sold by weight. They would full a complete article. Here are examples of each main type.


Figure 21: Fan, Toledo USA. From 1901
Figure 20: Plate, Stimson USA Pat. 1900



Figure 22: Cylinder, Toledo USA, from 1906


Figure 23: Disk,projected table, Avery 1960


## Conclusion

I suggest that Ready Reckoners and Tabular Calculators played a vital part in commerce for over 200 years. It is a pity that so few of the machines appear to have survived.
I would welcome information on any real examples.

## Acknowledgements

My thanks to the staff at the Science Museum (see more in Notes below) Peggy Kidwell at the Smithsonian Institution, Colin Barnes and Peter Hopp at the UKSRC, John Doran, Curator, The Avery Historical Museum (see Note), Dr Roger Johnson, my PhD supervisor at Birkbeck College, London, and many other patient and helpful correspondents who have answered my questions and supplied information and pictures.

## End Notes

i. Analysis of the On-Line BL catalogue, searching for Reckoner. Analysed in an Excel Spreadsheet.
ii. Warne's Large type Ready Reckoner, editions and reprints from 1927 to 1947, had "Calculations from 1/64 ${ }^{\text {D }}$. to $£ 1$... 60,000 calculations"
iii. See GB Slide Rules \& Calculating apparatus, Indexes to abridged Patent Specifications 1855-1963, by Bruce O .B. Williams, UKSRC 2002 ISBN953039 41
iv. From Thomas A Russo, Antique Office Machines, 2001, Schiffer Publishing Ltd PA USA ISBN 0-7643-1346-1
v. From the Catalogue of the Malassis-Chauvin collection, by Alain Brieux Paris, 1984
vi. Pictures from the Science Museum. I am most grateful to the Curator of Mathematical instruments, Jane Wess, and to Kevin Johnson for arranging to take special photographs, inside and out, of the Tabular Calculators in the collection. See also Calculating Machines and Instruments, Baxandall \& Pugh, The Science Museum1975
vii. Horsburgh, Napier Tercentenary Handbook, 1914, reprinted Tomash Publishers 1982
viii. Courtesy David MacFarland
ix. BOBW Collection Date uncertain
x. BOBW Collection Date uncertain
xi. Sources for Computing Scales include Equilibrium, the Quarterly Magazine of the International Society of Antique Scale Collectors. See www.isasc,org. Relevant articles in most issues since 1997 Issue2. Also the Curator of the Avery Historical Museum, John Doran Avery-Berkel Limited. See web site WWW.averyberkel.com

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${ }^{\text {vii }}$ Horsburgh, Napier Tercentenary Handbook, 1914, reprinted Tomash Publishers 1982
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${ }^{x}$ BOBW Collection Date uncertain
${ }^{\text {xixi }}$ Sources for Computing Scales include Equilibrium, the Quarterly Magazine of the International Society of Antique Scale Collectors. See www.isasc,org. Relevant articles in most issues since 1997 Issue2. Also the Curator of the Avery Historical Museum, John Doran Avery-Berkel Limited. See web site WWW.averyberkel.com.

## The Slide Rule Catalogue, its Past and Future

## Otto van Poelje

## Early beginnings

When the Dutch Circle of Slide Rule Collectors had only just been founded in 1991, Herman van Herwijnen was already plotting and planning his Slide Rule Catalogue. It was his project from the start, and he started with a considerable experience from the catalogue activities in old photo cameras, or Photographica, the area where he has learned the "art of collecting" during the 70's and 80's.
The well-known McKeown Guide for Cameras was the outstanding example at the time, but Herman had in mind to improve on such a catalogue, with the computer technology which was then becoming available to the public in the form of the IBM/Microsoft Personal Computer (PC) and the competing Apple Macintosh. Herman chose the Macintosh because it was more user-friendly, while most of the rest of the world became addicted to the PC, and that choice would make life not always easy when the Catalogue was later to be converted for the PC community.

## Database

Then another choice had to be made for the database, because the storage and handling of slide rule characteristics in a general database program was intended to be the major improvement over a paper-only catalogue. FileMaker Pro from Claris was chosen as it was the Macintosh flagship database system at the time.
A large number of more than 100 datafields was defined to cover all the aspects of a slide rule, from application areas,


Herman van Herwijnen, compiler and editor of the Slide Rule Catalogue physical aspects, scale design up to all printed matter on the faces of a rule. Exotic names like "RunnerType" or "Text slide front" (sbl/sbr/vertical) were introduced for unambiguous selection of datafields. And a lot of laws and rules were defined to ensure entering of characteristics of a slide rule in a consistent way. As an example the notation was introduced for specifying scales, like

$$
\mathrm{cm} / \mathbf{K} \mathbf{A}=\mathbf{B} \mathbf{C I} \mathbf{C}=\mathbf{D} \mathbf{L}
$$

for a common type of Rietz. Also many field values were predefined, so that drop-down menus could be used for data entry.
This however was mostly the concern of the person who maintained and expanded the Catalogue, which was Herman himself. He tried persistently to involve others in helping to enter data for their own collection, but most of the work was - and still is - being done by Herman.

On the other hand the user of the catalogue, accessing and reading the slide rule data, had to get into the FileMaker Pro program, which also became available for the PC around 1995: but typical PC users had problems getting used to the typical Macintosh user interface of FileMaker Pro.
For this reason the printed Catalogue of 1994 ("Blue Book", in two volumes) became the first real success of the project. But the Catalogue kept growing by the unrelenting efforts of Herman, pictures were inserted gradually, and updating a paper version (especially with more pictures) would become complex and costly.
Even loose-leaf systems were considered, but as quickly abandoned.
These historical events were addressed in some papers in IM1995 (Utrecht).

## Graphical revolution

Two happenings helped to advance the project further: the spreading of PC usage in the community of slide rule collectors, and the increasing availability of digital picture handling. At the moment that memory in Mega-Bytes was replaced by memory in Giga-Bytes, and also "burning" CD's became affordable, the computer 100 \% "Visual" Catalogue - with pictures - came within reach.
Herman started to take digital pictures of all existing and new slide rules in the Catalogue, a Herculean task, and this made the Catalogue infinitely more attractive to the users. All the text fields which used to supply position, direction
and fonts of imprinted texts, could be scrapped as soon as pictures appeared, because a picture gave that information a thousand times better. But still the text fields remain that can not be replaced by pictures, like dimensions or construction material.
In 1999 the Catalogue on CD-ROM was launched, see also Herman's paper in IM2000 (Ede).

## Today

The current Catalogue is larger than ever: over 3300 slide rules, each with multiple pictures, on just 2 CD-ROM's. Still another 2000 slide rules were described without picture. Many collectors have found the way to Herman, to order their own Catalogue CD's at 25.- EURO including postage.
In the first years only the two main collections in the Netherlands were covered; today many collectors from all over the world have sent their slide rule pictures to Herman for inclusion in the Catalogue, which in the early days used to be mainly Euro-centered.
When we look at the current database, we see more then 3300 JPEG picture files, to be accessed via the "run-time" version of FileMaker Pro which is included on the CD. This gives all the flexibility of the database function for lookup of data and pictures, selective access (like: show me "all Aristo's of plastic"), and reports of many "pre-cooked" formats.
The alternative use of the CD is by a picture browser (like ACDSee which can show many thumbnail pictures on a single screen), when folder and file organisation of the JPEG‘s is known: the key of any slide rule is the so-called Match number, a unique key which is embedded in the file names related to that slide rule (the NCV field, "Name Code - Variant" as explained at IM1995, is not used as main key anymore).


## The future

Gazing in the crystal ball, we can make some easy, and also some daring statements.

- We all hope that Herman will continue the good work of maintaining and expanding the Catalogue, please?
- Also we know that Herman is currently adding price data (and even "rarity") onto the Catalogue, doubtlessly influenced by his early McKeown example. This is interesting, as currently he follows prices paid at eBay auctions on internet over the last 4 years, as made by Ron Manley and Rod Lovett. But from history we have learned that catalogues can turn from "price followers" into "price setters"!
- Expansion of the Catalogue could be done in more ways than the contributions by fellow collectors. For example, the eBay auctions sometimes show rare or unusual slide rules that could be added if description and pictures are acceptable.
- Of course the Catalogue will have to get on DVD soon (who wants a pile of CD's?).
- A very important expectation -just need somebody to do it- is making the Catalogue available on internet. The Dutch Circle's website is an obvious candidate, but struggles with memory and consequently financial constraints (more than 1 Giga-Byte would be needed on the server!).
Maybe some solution will be found ....?
- FileMaker Pro is still alive and kicking, and currently it is possible to download FM Pro databases (or have wireless access) in PDA's: Personal Digital Assistants, or "organisers" like the Pocket PC. Thus every collector will have the complete Catalogue at his fingertips while browsing the flea-markets of the world.

I leave it to the reader to distinguish the easy from the daring forecasts, and to help realize them all! In the meantime, the current Catalogue is the best companion for every collector to provide data and pictures of all known slide rules!

## Dutch Architecture during the Initial Period of the Slide Rule

## Henny C. Brouwer

## $16{ }^{\text {th }}$ century Scientific world

Renaissance deals with the rebirth of man and world and caused a revolution in the scientific and artistic $16^{\text {th }}$ century West European world. No longer the church was master of the human mind, it was the Antique world and the great scholars like Aristotle and Plato, who inspired the renewed spiritual, scientific and artistic life. The fascination for the microcosmos as well as for the macrocosmos authorised the development of a range of physical and mathematical instruments - like the microscope and telescope - in order to study, research and understand life, earth and universe. This new intellectual approach led to long lasting conflicts and processes between scientists like Copernicus (1473-1543) ${ }^{1}$ and Galileo (1564-1642) ${ }^{2}$ at one side and the Church and State at the other side, sometimes with a disastrous ending.

## Artistic world

In this turbulent world, the anonymous devoted craftsmen of the medieval workrooms changed to well known artists with their own personal style and signature! Not only the bible was their inspiration (religion never stopped inspiring), but also the classic world, daily life and nature itself and the traditional static, almost frozen gothic figures - in sculpture as well as paintings - suddenly started to move.... Important renaissance artists are Rafaël, Botticelli, Da Vinci and Michelangelo.

In architecture the ancient classic architecture became most important. The traditional vernacular architecture, based on local building materials, did not satisfy the educated class anymore. Several $16^{\text {th }}$ century Italian writers, Serlio, Palladio, Vignola and Scamozzi (Figure 1) reinterpreted the Ancient architecture of the Greek and Roman civilisation and published their personal views on mathematical proportion systems and related detailing, based on the different architectural styles. Which are these styles (Figure 2)?


Figure 1: Scamozzi Ionic


Figure 2: Five architectural styles

Two schools represent the old civilisation of Greece, the $5^{\text {th }}$ century BC: the Doric style and the Ionic style, geographically seen, the east and the west, separated by the Aegean Sea. The severe Doric style mirrors the robust

[^0]taste of the peasants of the continent in the west, the more elegant Ionic style - characterised by the typical scrolled capital - mirrors the complex and rich taste of the cosmopolite merchants of the islands in the east, highly influenced by their contacts with the older civilised cultures of Asia. The third style, the most refined and perfect Corinthian order - characterised by the acanthus leaf - dates from 100 BC and is considered to be a Greek, but not a classic Greek style.

The Romans, rationalists who ruled out the ancient spiritualism in their occupied areas, were excellent engineers well known for their constructions like arches and vaults, canals, bridges and aquaducts, their masonry and concrete. They found their architectonic inspiration in the Greek architecture, not just by copying this, but by using the architectural elements in an unconventional way, best shown by the Coliseum in Rome (80AD) in its different columns piled up to different levels, from the basic robust Tuscan order (a Roman variation of the Doric style), via the elegant Ionic order and the refined Corinthian order, to what they considered to be the best of all on top: the Composite order, a composition of Ionic and Corinthian elements, extremely refined, but not quite a brilliant new design.

The Roman empire fell in the $5^{\text {th }}$ century, but their rationalistic way of thinking is still ours..... Their architecture survived as the buildings could be reused for other purposes, but many Roman temples only provided useful building material for - amongst others - the famous early Christian churches in Rome.....

The Italian books on architecture, published in the 16the century, circulated all over Europe. Serlio's " Five Books of Architecture" were published in 1537 and translated in Dutch in 1539 by Pieter Coecke van Aelst. In 1555 Hans Vredeman de Vries published a version with large engravings that clearly showed the construction of the perspective and became most popular in the workrooms, especially for painters (Figure 3).

Serlio's first book of Architecture dealt with Euclides' mathematical system. Mathematics were not exclusively reserved for scientists only, in order to explore the world, but became an indispensable part of education for tradesmen, clergymen, navigators, military men and architects as well. Seen in this light it is not surprising that in this period instruments for accurate measuring and calculating were developed, such as - for instance - the pendulum clock (Christiaan Huygens 1657) (Figure 4), the proportional compasses (?), and the sliderule (Oughtred 1620).

For architects the art of geometric proportion meant a way to create beauty and order. In Serlio's fourth book all this was explained by the square, the diagonal, the inner and the outer circle. During the early Renaissance period the decorative ornament seemed most important, later on the ideal mathematical system of the classic proportions introduced a more balanced classic style: the $17^{\text {th }}$ century Dutch Classicism. At the end of that century all decorations disappeared and the utmost balanced, sober and flat Classicism was introduced.


Figure 3 : Perspective


Figure 4: Haags Klokje of Ch. Huygens

## Holland in the $\mathbf{1 6}^{\text {th }}$ century

The Renaissance started in the Netherlands in the second half of the $16^{\text {th }}$ century in a period in which the country fought for its independence ( $1568-1648$ ) and in many ways broke with the past. In the fairly political stability of the beginning of the 17 the century the Dutch republic was founded as a republic of burgers (citizens)! This fresh approach provided economic, scientific and cultural expansion, highly influenced by well schooled and experienced merchants refugees from the south, in particular from Antwerp and rich Portuguese Jews. In this "climate", with freedom for religious and political conviction, more books were printed in the Netherlands than in the rest of the world ${ }^{3}$.

## Holland in the $17^{\text {th }}$ century

The small seafaring nation, using small manoeuvrable ships, and trading in the German, Scandinavian and Baltic region, now started exploring the world and found itself a regular way to Asia, by forcefully taking over many Portuguese settlements in Africa and Asia.

In 1602 in Amsterdam the first multinational in the world - with shareholders from all layers of the population ${ }^{4}$ - was founded in Amsterdam: the Dutch East India Trade Company (V.O.C.) and made it the most important city in Holland, although The Hague was (and still is) the Residence. Harbours, docks and warehouses named after foreign cities mirrored the international contacts.

Spices, especially the very expensive cloves from the Moluks and cinnamon from Ceylon as well as cotton cloths from India became important products for trade. Chinese tea found its way to daily life and Chinese porcelain became highly collectable and exclusively exhibited in the private reception rooms. Books were published about foreign life, vegetation and animals and cabinets full of exotic artefacts, shells, coins, ivory, etc. became highly fashionable and showed off the international interest of the Dutch "burger".

The tulip, originated from Persia and introduced in Holland by the botanist Clusius in 1593 as a curiosity , became so popular during the first half of the $17^{\text {th }}$ century, that even a settled physician as Dr Claes Pietersz., pioneer in pathologic anatomy and portrayed by Rembrandt van Rijn, named himself after his house on the Keizersgracht in Amsterdam: Nicolaes Tulp. Extremely high prices were paid for new varieties, the unlucky rage finally ended in a financial catastrophe in 1636.

## $17^{\text {th }}$ century Amsterdam

The city of Amsterdam started as a $13^{\text {th }}$ century settlement around a dam in the river Amstel, protected by dikes (Haarlemmerdijk, Zeedijk and Hoogte Kadijk, where we have our International Meeting 2003) against tidal water of the Zuiderzee and surrounded by an wooden rampart.

The first buildings were made of timber - Holland means "houtland", woodland - but large fires in the 15 the century initiated municipal proclamations to build in brick and it may be surprising that many old wooden houses still exist in the city, only masked by a modern brick exterieur. During the $15^{\text {th }}$ century the eggshaped oval town, halved by the Amstel, was provided with a brick fortress with gateways and bastions (figure 5).

[^1]

Figure 5: Map of Amsterdam 1544


Figure 6: Map of Amsterdam 1795

The economic prosperity and increasing population at the end of the 16th century made a new extension necessary, in eastern direction. In 1609 the city council decided to expand the old city by the - now well known - concentric circular canals, from inner to outer circle, named Herengracht, Keizersgracht and Prinsengracht - protected by new fortifications. The execution of this ambitious plan, that started against clockwise, seen from the mouth of the Amstel into the Zuiderzee, took nearly the whole century (Figure 6).

During its construction the well-to-do citizens started to move from the noisy and crowded docks and warehouses in the centre to the fashionable and spacious houses that were built along the new canals. In the overall view of this new mathematical outlay two area's attract the attention: the Jordaan, in the west, and the "islands" of Uilenburg, Rapenburg en Marken in the east, which follow the quite different, already existing pattern of drainage ditches and pastures of the land. These area's housed the smaller tradesmen.

The V.O.C.'s main settlement was located north of the old (Hoogte Ka-) dike along the Zuiderzee, on one of the new islands, called Oostenburg. Here the company had its own ship-building yards, docks and warehouses and housed its workers. From here prosperity came over the Dutch Republic until the economic recession in the $18^{\text {th }}$ century caused by several wars - and finally its closing down in 1799. The enormous main building of the V.O.C. was reused afterwards for grain storage until it quite spectacularly collapsed in 1822.

The area kept its shipyards, industries and warehouses until the second half of the 20th century. The general interest for industrial archaeology today protects what is left over of the buildings, sites and the characteristic atmosphere and the warehouses seem to fit extremely well for living.

## $17^{\text {th }}$ century townhouse, plan and interior

The $17^{\text {th }}$ century houses along the Amsterdam canals can be divided in two types, the narrow house and the wider house.

Since the $14^{\text {th }}$ century building-lots of 30 Amsterdam feet were common and for the first half of the new canal-ring one did not break with this tradition, which meant long narrow houses, only now with the traditional spiral staircase modernised to the wider type in a more prominent position along the corridor between the front-house (voorhuis) and the rear (achterhuis).

The facade of the narrow house traditionally has three bays, the main door is situated left or right, on the first floor above the street. The two en-suite rooms in the front house, although representative, were inferior to the receptionroom or great-hall (zaal) in the rear. The kitchen was situated in the basement in the rear.
The entrance hall used to have a stone floor, a beamed ceiling and plastered walls, mostly furnished by maps. The en-suite rooms would have wooden floors and hangings, woollen or guilded leather if the room was used as dining room. Paintings, landscape, biblical scenes, classical scenes or family portraits (painted by Jan Van Gooyen,

Ferdinand Bol, or Rembrandt van Rijn (Figure 7)) were standard. The reception-room might have a painted decoration on the ceiling and Flemish tapestry hangings.

For the second half of the canal-ring, executed during the second half of the century, a double parceling was chosen. This wide house with preferably 5 bays had the main entrance in the middle, again on the first floor above the street. Although the house, like a countryhouse, seemed symmetrical, it nearly always had a strict asymmetrical plan, which divided the house in a private and a reception area. The private room was situated beside the main door, next to the main staircase in the centre of the house.

The other rooms were connected in a hierarchical order, reflecting the French apartment: "antichambre", "chambre" (the en-suite rooms) and the "Salon" with a view of the garden, so providing a varying and impressing "axis of honour" for important guests. The kitchens and storage rooms were situated in the basement, the family lived their daily life on the second floor, the servants slept at the attic. In the interior decoration the wider house would be similar to the narrow house, only the grandeur was more overwhelming.


Figure 7: Rembrandt van Rijn


Figure 8: Facade proportion Schielandhuis 1662

In general one could say that during the winter - with very few theatres in Amsterdam - social life of the upper class was completely concentrated at home and it seems quite obvious that the house, its interior decoration and the host's exotic collections played an important part in entertaining family and friends. In the beginning of the summer one left the warm and dusty city by coach or trackboat for the country. A countryhouse, preferably along the river Amstel or Vecht, near Maarssen and Breukelen (Brooklyn) - where we have our International Meeting 2003 - would provide the citizens the necessary change of entertainment.

## Architecture

The most important architects in the early 17 th century in Amsterdam were Hendrick de Keyzer (1565-1621), the painter/architect Jacob van Campen (1596-1657) and Philips Vingboons (1607-1678).

Their designs - frequently represented in the first half of the canal circle - show red brickwork facades with yellow sandstone embellishments in late Renaissance style as well as Classic sandstone facades with pilasters, cornices and pediments, following a basic proportion-scheme of circles, squares and diagonals ${ }^{5}$ (Figure 8) and perfectly detailed in conformity of the Books of Architecture of Scamozzi, in only a three bay house!

Recent studies pointed out that even the ground plan of many designs was based on these mathematical elements, although in practice this was hard to realise dealing with the personal wishes of the commissioner and the capabilities of the site. Adriaan Dortsman (1635-1682) was the most important architect in Amsterdam at the end of the century. At that time the decorative classic pilaster-facade was over. Dortsman's work represents the Sober Classicism, that was most fashionable during the executing of the last parts of the canals. Characteristic for this

[^2]classicism is the almost flat architecture, crowned by an elegant balustrade, in perfect proportion, with very few windows, almost introvert in comparison with the earlier expressionistic facades.

## Conclusion?

In the 17th century the technical development as slide rules, clocks, etc. resulted in inventions and created trade, which generated financial possibilities for culture.
Many $17^{\text {th }}$ century houses of architectural importance, along Herengracht, Keizersgracht and Prinsengracht, although more or less modernised to please and comfort later inhabitants (Figure 9), still exist. Their proportions seem so well balanced that they are easy to recognise and still please the "connoisseur". Is this evident or just a coincidence?


Figure 9: Van Loon Staircase

## Sources

Christiaan Huygens, Museum Boerhave
Museum van Loon information booklet
Pieter Post by J.J. Terwen \& K.A. Ottenheym
Perspective by H. Vredeman de Vries
Philips Vingboons by K. Ottenheym
Gli Ordini Classici in Architectura by R. Chitham
A Humanist Prince in Europe and Brazil by E. van den Boogaart
De zeventiende eeuw. Het Hollandse wonder by H. de la Fontaine

## Slide Rules for Metal Workshops

## Gerard van Gelswijck

## Introduction

When, many years ago, I started my education in the machining of metals as an apprentice (fig. 1) at one of the largest Dutch engineering firms, Werkspoor, later amalgamated with Stork, I was soon very curious about one of the tools the planner used. I was told that this device was a "rekenliniaal" (slide rule), a word which, at that time, had little meaning for me. A few years later, as I was trained as an engineering draughtsman, which also included some lessons concerning theoretical subjects, I got my introduction to the use of the slide rule with the help of the well-known ARISTO instruction slide rule. And very important, I obtained my first slide rule, also an Aristo. This and the possession of a nice set of drawing instruments of Wild manufacture filled me with pride and made me feel well-qualified for a career in

figure 1: Author's apprenticeship mechanical engineering. That career is now completed but my work was also a kind of hobby for me and therefore I stand today before this audience to tell you something about the art of cutting metal and the uses of the slide rule in this branch of mechanical engineering.

## Machinetools

First of all it seems useful to give an introduction into this profession and show you the more common ways of machining metal as used in the workshops of thirty or forty years ago. For nowadays the principles of machining metals are of course the same, but the evolution of

figure 2: Principle Parts of the Lathe
quality.
In the first place it must be clear that in every cutting process there are two distinct motions:

- The cutting motion
- The feeding motion

Take for instance the sharpening of a common lead pencil with a penknife: first you put the knife on the pencil in such a way that you can take away a chip of wood and then with a sliding movement you cut off a chip, and you repeat this process until the pencil is sharp. Your placing of the knife on the pencil, in machining led to a situation in which the function of several machine tools is combined in a so called Machining Centre which is capable, with the aid of a computerised control system, to turn out a flow of products of a constant and high

figure 3: Milling Machine
fact your decision about the thickness of the chip, is called the feed, and the sliding motion is called the cutting. A classification of machine tools can be made along these two motions, and so we distinguish the lathe (fig. 2) in which the cutting motion or main motion is performed by the rotation ("Turning"), in the headstock of the spindle A on which the face-plate C is mounted which in its turn carries the work-piece, and the cutting tool, fastened in the

figure 4: Drill with Jig Vise
 tool-holder O which exerts the feed.
In the milling machine (fig. 3) the cutter revolves and the work-piece, mounted on the table, is fed towards this rotating cutter. Drilling is another chapter, for it is possible the let the drill do both motions, cutting and feeding, on a drill-press or other type of drilling machine (fig. 4), but it is also possible that the work-piece rotates and the drill is fed in; this is normally the case with drilling on a lathe.
Yet another way of machining metals is planing and shaping (fig. 5 and 6). In the above-mentioned processes the cutting motion or main motion was a circular one and the feed a linear one, in planing and shaping both of them are linear ones and the feed is also intermittent instead of continuous.
Nowadays planing and shaping is largely replaced by milling, mainly for two reasons:
Due to the fact that the cutting motion is an linear one, the cutting stroke must be a little more than the length of the work-piece and is followed by a return stroke without cutting, which is very inefficient
At the start of the cutting stroke the cutting tool is advanced a certain distance, the feed, and as a consequence the tool hits the work-piece with considerable force. Therefore a shaping or planing machine must by sturdily built, and the reversing of the large moving masses of table (part 25 in fig. 5) and work piece in a planing machine (fig. 5), or the ram with the cutting tool in the shaper (fig. 6), which must be reversed after each cutting stroke and of course again at the end of the return stroke to start a new cutting stroke, requires a lot of power. All these factors led to the demise of these machine-tools and the growing use of milling machines.

figure 6: Shaping machine

## The use of slide rules

We are now familiar with the most popular machine tools, but are wondering about their connection with the slide rule. This connection is very simple for every manager tries to minimise his costs and in doing so maximise his profits, so it is unavoidable that he must make some calculations to determine, among other costs, the production times of the parts of his product, after this he is also able to give an estimate of the delivery time. In general all necessary calculations are of simple nature and can be performed on a normal slide rule of the Rietz type, but some
firms have developed special types. Let us have a closer look at the calculations and the several factors involved. In the main there are three different calculations:
a) Calculation of the machining time
b) Power required for the machining
c) Conversion of cutting speeds into revolutions

## Ad a) Calculation of the machining time

A calculation for determining the time required for the execution of a certain machining operation, and for this the general formula is: length to be machined divided by the feed per minute gives the machining time. In turning, the feed depends on the number of revolutions of the work-piece per minute, for the feed is expressed in mm per revolution and the number of revolutions depends on the permissible circumferential speed or cutting speed. In milling, the feed is expressed in mm per minute using the formula:

## Feed =

feed per tooth * the number of teeth of the milling cutter * the number of revolutions per minute of the cutter.
The cutting speed (the circumferential speed of the cutter) " $v$ " depends on the material to be machined and the material the cutting tool is made off. After numerous experiments with all the possible combinations of materials, the manufacturers published the found data in the form of tables and recommendations. The cutting speed " $v$ " is, in most cases, the mean time between re-sharpening of the tool, during which the tool gives an even result and this time is standardized as 60 minutes. Very soon the researchers discovered that there is no linear connection between the cutting speed and the mean time between re-sharpening of the tool, with other words halving the cutting speed does not double the mean time between re-sharpening. Instead they found the relation $v \cdot T^{n}=C$. This expression is known as Taylor's Rule, after Frederic Taylor, an American engineer who -a hundred years agostarted the systematic research in the field of the machining of metals. A graphic representation of this expression is given by fig. 7. Another factor with a great influence on the cutting speed is the material of which is the cutting tool is made. The two most

figure 8: Hardness of Materials used are High Speed Steel or HSS ("snelstaal" in fig. 8), and Tungsten-carbide ("hardmetaal" in fig. 8) for special purposes, yet other materials such as diamond and ceramics are used, but in normal workshop practice you do

figure 7: Taylor's Rule encounter them not very often. HSS was developed by Taylor and its main advantage is the fact that it retain its hardness at higher temperatures than the harder plain carbon and tool steels, see also fig. 8 in which can be seen that Tungsten-carbide still has a useful hardness at about 1050 degrees Centigrade. This property made it possible to use in machining with Tungsten-carbide cutting speeds 3 to 5 times higher, depending on the material to be cut: higher than when using HSS. This fact came as a nasty surprise to the British at the start of the Great War, for they could not believe that the Germans were able to produce shells at the same rate as they fired them, but the Germans did this thanks to Krupp who had developed and produced the first practicable form of Tungsten-carbide, better known as WIDIA, an abbreviation of (hart) WIe DIAmant.

## Ad b) Power required for the machining

Having said something about cutting materials and speeds we come to another point in which a planner is interested, namely the power required for a specific operation, for the available power of a machine tool is limited. The formula used says that the power is:

Power in kW =
Cutting speed ( $\mathrm{m} / \mathrm{s}$ ) * the specific cutting force of the material to be machined * area of the chip

figure 9: Point of Drill

This figure is the power required for the cutting, the installed power must be higher depending on the efficiency of the transmission of the machine tool, it varies between 75 and $85 \%$. I remember that in the engineering works in which I served my apprenticeship, all machine tools were equipped with ampere-meters with a red line at the maximum permitted current and thus we, juniors, were instructed at all times to try hold the hand of the meter near that red line. In all metal machining processes there is no need to calculate the power for the feed motion except for drilling, as everybody knows who ever tried to drill a 10 mm hole in stainless steel with an electric hand drill.
A look at the point of the drill (fig. 9) gives us an explanation for this phenomenon. To give the necessary rigidity to the drill, it has a core with very bad cutting properties, in fact you must press this core, so to say through the work-piece and

figure 10: Drill feed and power this requires brute force, the force you have to apply on your 10 mm drill, in trying to drill a hole in that piece of stainless steel. Good practice therefore is to pre-drill that hole with a much smaller drill to get good results, see fig. 10 for actual values of this feeding forces.

## Ad c) Conversion of cutting speeds into revolutions

The third formula is used for the conversion of cutting speeds into revolutions or strokes per minute and it sounds as follows:

## cutting speed $=$

$\boldsymbol{\pi}$ * diameter of work piece or milling-cutter * number of revolutions per minute
Now we have seen that the formulas used by the planner for his calculations are of a simple form and can be easily performed on a simple slide rule and with the help of tables and so it was done for many years. But around 1930, Nestler came up with their model 0260 especially designed for the planner of operations on machine tools.


Rüdseite der Zunae
figure 11: Nestler 0260-Mecanica

It was a versatile instrument based on the results and recommendations of the "Ausschuss für wirtschaftliche Fertigung" (AWF), which means "Committee for efficient Production".
A few weeks ago I was asked by the curator of the Werkspoor Museum whether I knew which kind of slide rule it was, which she recently had been given by the son of a former employee, it was a well-worn Nestler 026. With this type of slide rule and the later model 0260 Mecanica (fig. 11), which is a combination of the type 26 and the type 30 G intended for the calculation of the weight of bars, sheets and pipes in the regular dimensions, a planner was able to do, with the help of tables, the biggest part of his work.
An example of a possible calculation is shown in figure 12.1 and 12.2 . On a lathe with a 6 kW motor one wishes to produce axles from mild steel with a tensile strength of $42 \mathrm{~kg} / \mathrm{mm}^{2}$, the depth of the cut is is 4 mm and the feed is 0.3
$\mathrm{mm} / \mathrm{rev}$. Question: What is the maximum cutting speed?
Solution: Set the feed of 0.3 mm on the green scale of the slide below the depth of the cut, 4 mm , on the top left green scale of the stock of the slide rule.

figure 12.1: setting of feed and cut depth

figure 12.2: setting of cutting force

Place the cursor on the mark kW , pull the specific cutting force (42, taken from the table on the back of the rule), below the hairline, and read, under 6 kW on the upper scale of the stock, the resulting value of $142 \mathrm{~m} / \mathrm{min}$ on the upper scale of the slide. In laying out the scales of this slide rule, a mechanical efficiency of $75 \%$ has been assumed, which is fairly low. Besides Nestler there were other firms who marketed slide rules intended for use in workshops, but as it was a specialised field, there were only a few other manufacturers: well known on the Continent were the Sonderrechenstäbe of the AWF, see fig. 13.

figure 13: front and back of the AWF slide rule

## Slide charts

Besides the real slide rule much used was the slide-chart, often supplied by the manufacturers of machine- or cutting tools such as drills and mills as a kind if promotion material. In most cases they are a handy forms of tables containing conversions from cutting speeds to rpm, or cutting time in relation to the feed rate and so on. Manufacturers also supply discs with instructions for the selecting of the proper cutting tool or material grade in relation to the material to be cut and the desired finish. These charts and discs are often very useful but sometimes they must be used carefully.

figure 14: Filetta slide chart
For instance the Filetta slide-chart (fig. 14) was much used in Holland, but due to the change-over from DINstandards to ISO-standards you can not longer trust the recommendations given for dimensions of the holes for tapping the screw threads, they are in general to small and will give a lot of trouble, in the form of difficult to remove broken taps.

## Special slide rules

For the calculation of machining times on the common machine tools, there were enough tools in the form of slide rules, tables and monographs available, but sometimes the need arose for a slide rule for a specific purpose or machine. One of the difficult calculations in using the machine tool, especially the lathe and the milling machine, was the calculation of the change wheels needed in screw cutting or dividing (fig. 15). In every textbook dedicated to turning and milling, many pages are used to explain the methods used. The problem seems simple, in using the lathe we have seen that the feed is expressed in mm per revolution of the work-piece. It is therefore easy to understand that the feed-shaft is driven from the main-spindle. In simple lathes, the feed-shaft doubles as lead-screw (see part K in fig. 2), in bigger lathes there is a separate lead-screw which is also driven directly by the main-spindle. Thus by changing the gear-ratio between mainspindle and lead-screw the feed is also changed. In doing so one can select for

figure 15: Change Wheels for Screw Cutting instance a feed of 1.5 mm per revolution and obtain with the proper cutting tool a V-groove or a form of screwthread. For the commonly used screw-threads you usually find some kind of table attached to the headstock of the lathe, but there are numerous screw-threads possible, so are the gear ratios between main-spindle and feed-screw.

But there is only a limited set of gearwheels available, so for the uncommon threads one must carefully calculate the needed set of gears. Often the manual supplied with the lathe, gives all the combinations possible with the set of gearwheels available. A particular problem is the cutting of screw-threads with metric dimensions on a lathe with Imperial dimensions or vice versa. This problem can be completely solved by including a gearwheel with 127 teeth in the set of gearwheel, for 5 inches are 127 mm , a fair approximation can be reached by using a gearwheel with 63 or 64 teeth. So if you see a lathe or milling machine and you discover a 127 teeth gearwheel and you say:" Oh, look a 127 wheel, this is a universal machine", everybody thinks you are an expert. The already mentioned Werkspoor museum has in its collection a special slide rule designed to solve the above described problems, especially when one must cut spiral grooves with a big lead. This is usually done on a milling machine, because in a lathe the lead-screw is driven from the main spindle and this put a lot of strain on the transmission. This particular slide rule was designed and used in the "Gereedschapmakerij" or Toolroom, in this department the most skilled craftsmen are employed, mostly in making special tools such as drills, milling cutters, taps, dies and reamers, for use by the production departments. Its design is probably based on the following thought: Gear-trains usually consist of four wheels, two being drivers and two being driven, either of the drivers is fixed on the main-spindle and gears one of the driven, which is placed on a adjustable intermediate shaft or stud. The second driving wheel is connected to the first driven and gears with the remaining driven which is placed on the lead-screw. An example will clarify this case: What gears are required to cut 24 threads per inch on a lathe with a lead-screw having 2 threads per inch. The gear ratio is $2: 24$, the smallest gear is usually 20 , this would mean that for a simple train gears of 20 and 240 would be required. A gearwheel of 240 teeth is not available so we must use a compound gear train with 4 wheels, which train is calculated as follows: $\frac{20}{240}=\frac{2}{6} * \frac{10}{40}$ in other words, the numerator and the denominator have been split up into their fractions, now the numerator and denominator of $\frac{2}{6}$ are multiplied by 10 , and the numerator and the denominator of $\frac{10}{40}$ by 3 , and in doing so we get the following combination for cutting 24 threads per inch:

$$
\frac{\text { Drivers }}{\text { Driven }}=\frac{20}{60} * \frac{30}{120}
$$

We can write this formula in general form $\frac{A}{C} * \frac{B}{D}$ and then calculate all combinations of $\frac{A}{C}$ and $\frac{B}{D}$ possible
with the set of gearwheels supplied with a particular milling machine. The numbers found are engraved, on a logarithmic scale on the places of the A and B scales on an ordinary slide rule and on the C and D scales are engraved the diameter of the work piece and the angle $\alpha$. This angle is the angle of which the tangent is:

$$
\frac{\pi * \text { diameter of workpiece }}{\text { lead or pitch of the spiral groove }}
$$

The angle $\alpha$ is the angle to which the table of a milling machine must be set for cutting spiral grooves into a cylindrical work piece (fig. 15). On the slide are engraved two logarithmic scales for the leads in mm's and inches, one in red for values from 10 to 240 mm or 0,5 to 9,5 inches corresponding with the red hairlines to the right side of the slide rule. The black scale runs from 240 mm or 9,5 inches to 6250 mm or 250 inches, and these are used in conjunction with the black hairlines on the left side. In cutting this kind of spiral grooves, an example can be found in the tiny spiral-grooved spindles of the famous Curta, on a milling machine an indexing or dividing head ("verdeelkop" in fig. 15) is always used. The internal gear-ratio of the worm/wormwheel transmission is 40:1. This means that if the lead-screw ("draadstang freestafel" in fig. 15) of the table of the milling machine drive the worm of the dividing head with a gear ratio of $1: 1$ and the lead of the leadscrew being 6 mm , you must turn this lead-screw 40 x to turn the work piece mounted on the main-spindle of the dividing head 1 x and after this turning of the screw the table of the milling machine has moved $40 * 6=240 \mathrm{~mm}$ or in the case of a British or American built machine with a lead of $1 / 4$ of a inch $40 \times 1 / 4 "=10$ inches or 254 mm . With other words you have cut a spiral groove or helix with a lead of 240 or 254 mm . By changing the gear-ratio between lead-screw and worm-shaft you can get other leads and so you can use this slide rule to calculate the gear-ratio needed for a certain lead on the scale on the A and B position and the angle $\alpha$ on the lower half of the slide rule. The figures 16 and 17 give some impression of this rare instrument.

figures 16 and 17: The unique Werkspoor slide rule

## Conclusion

As this lecture was not intended as a treatise on Turning or Milling of metal but aimed to give you some insight in the use of slide rules in production engineering, I hope that it has served its purpose, and for me it is a pity that with the demise of the slide rule also large scale Engineering Works and Shipyards have disappeared.

## Acknowledgements

I wish to thank Mrs. Drs. Heleen Stevens, curator of the Werkspoor Museum, for the opportunity given for taking photographs and displaying the special Werkspoor-made slide rule.

## Blbliography

[1] Vervaardigingskunde, Ir. H.J. Hovinga/ing. A.J. Deurloo, 1988, T.U. Delft
[2] General Engineering Workshop Practice, several authors, 1959, Odhams Press Ltd, London
[3] Gereedschappen, Th.T. Hermans, P. Out N.V., Koog aan de Zaan
[4] Advanced Machine Work, Robert H. Smith, 1925, Industrial Education Book Co., Reprint
[5] De Beginselen van de Gereedschapwerktuigen en de Metaalbewerking, Ir. G. Hofstede, 1950, Æ.E. Kluwer, DeventerDjakarta

## UTO Manufacturing Process

## John Kvint

Description based on manufacturing a special slide rule for X-ray exposure calculations.
The slide rule has 2 slides with functions on both sides.
The slide rule is 181 mm long, 43 mm wide and operates with one single-line cursor.


Figure 1: The X-ray Slide Rule

1. Process: Sawing rectangular sheets for body, using 4 mm white press-polished hard PVC.

Formerly this was done using an ordinary carpenters rotary saw.
Lately the sheets are cut by the Danish importer of the PVC plates.
2. Process: Sawing rectangular sheets for slides, using 2 mm white press-polished hard PVC.

Formerly this was done using an ordinary carpenters rotary saw.
Lately the sheets are cut by the Danish importer of the PVC plates

Figure 2: Silk-Screen Printing of the 7 Bodies

3. Process: Silk-screen printing 7 front-bodies on one sheet, every body has 2 scales. Black ink. Formerly UTO used a complicated photo-etching process.
Later there have been developed printing inks that adhere to PVC, well enough to replace photo-etching.
Using a high-definition capillary stencil film, 0.1 mm division lines are printed together with indications of 1.1 mm height.

Figure 3: Control of Alignment, Front to Rear

4. Process: Silk-screen printing 7 bodies, reverse side of body sheet. Black ink.

As process 3. And with very careful alignment to the front.
5. Process: Silk-screen printing 9 pairs of slides on one sheet, every slide has 2 scales. Black ink. As process 3. Special care to print on the thinner material.
6. Process: Silk-screen printing 9 pairs of slides on the rear slide sheet. 2 scales per slide. Red Ink. As process 3 . With very careful alignment to the printing on the front side.

Figure 4: Silk-Screen Printing of the 9 Slides

7. Process: Control of silk-screen printing. Marking items to be rejected during further processing.

Printing quality studied using a magnifier. Marking with permanent inkpen.
8. Process: Using rotary saw to cut bodies to final length, 181 mm .

UTO saw nr. 1. Belt-driven for smooth running. Blade thickness 0.8 mm . Clamping one sheet at a time, the saw is spindle-fed and cuts automatically.

Figure 5: First the Bodies are cut to Length

9. Process: Using rotary saw to cut slides to final length, 181 mm .

As process nr. 8 .
10. Process: Using rotary saw to separate the bodies.

UTO saw nr. 1. The sheets are aligned according to printed indications and clamped.
11. Process: Using 4-bladed rotary saw to separate the slides.

UTO saw nr. 2. The blades are positioned to cut one pair at a time from the sheet.

Figure 6: Cutting off one Pair of Slides at a time

12. Process: Shaping the bevel and two bottom grooves, which makes the body flexible.

Shaper nr. 1. Special belt-driven design by Larsen to avoid rattle-marks which could


Figure 7: Shaping Bevel and Flexible Bottom
occur using the original gearbox. Special tool-holder for multiple chisels.
A: Shaping the bevel surface with 8 degree inclination.
B: Shaping the upper bottom channel.
C: Shaping the lower body channel.
Automatic down-feed of tool-holder using a ratchet mechanism built into the manual feed-handle.
13. Process: Milling upper groove for the cursor.

A small single spindle router is used.


Figure 8: Cutting of the Upper Cursor Groove
14. Process: Milling the lower groove for the cursor. As process nr. 13.
15. Process: Routing the central channel using a multiple rotary tool, that cuts the channel-bottom, and the upper and lower slide keyway in one operation.
A special router with a sidewise-sliding motion. The special tool-holder accepts 4 small tools that during the rotary motion cut the unique channel profile.


Figure 10: Mechanically fed Router cuts the Central Keyways
16. Process: Shaping the body using a multiple planer tool that performs the following operations:
a: Shaping the edge of the measuring scale bevel.


Figure 11: Multiple Planer and Finished Ruler Profile
b: Shaping the bevel surface with 8 degree inclination (last stroke only).
c: Shaping the vertical and bottom surface for the upper slide.
d : Shaping the vertical and bottom surface for the lower slide.
e: Shaping the upper channel bottom for body flexibility (last stroke only).
f: Shaping the lower channel bottom for body flexibility (last stroke only).
g : Shaping the lower edge of the body.
Shaper nr. 1. An additional tool-holder allows two layers to accommodate up to 8 chisel-like tools.
17. Process: Press-cutting finger cut-outs in both ends.

Using the die press for cutting finger cut-outs on pocket slide rules, one has to relocate the body sidewise to obtain a wider cut-out for the pair of slides.
18. Process: Routing rounded corners in both ends.

This is a copy-milling process, first one end, next the other end placed upside down.
19: Process: Manual de-burring finger cut-outs
The sharp cut edges are rounded using a de-burring tool.

Figure 12: Rounding Corners at Left End

20. Process: Manual de-burring. Generally 45 degrees chamfering overall.

A special tool made from a hacksaw blade is ground to contain several useful shapes.
The thickness of the tool ensures uniform cuts. The handle is wound using textile tape.
21. Process: Manual smoothing the slide keyways.

Small chips and dust may remain in the keyways. A brass tool with the profile of the key on the slides is passed through the grooves. Cuts in the tool brings the dust out in the same manner as the teeth of a saw.


Process 22: shaping slides

Figure 13 a and b: Shaper for 2 Slides at a Time
22. Process: Shaping upper and lower edge of slides, using a double tool.

Shaper nr. 2. Equipped with a pair of horizontal clamps to hold a pair of slides.
First tool cuts the sliding surface between upper and lower slide.
Second tool is a pair of chisels, cutting the key upper and lower side simultaneously.
10 pairs made and checked sliding in one body.
23. Process: Chamfering ends of slides.

Pushing the slides against an adjustable stop on a small rotary planer makes a chamfer.

Figure 14: Chamfering Ends of Slides

24. Process: Manual de-burring ends of slides.

The same hand tool as used in process 19 .
25. Process: Hot foil imprinting of 7 inch scale on the bevel.

Using a bookbinders heated press, the bodies are placed on an underlay angled 8 degrees to bring the bevel into a horizontal position. A heated magnesium die-stamp is pressed against the bevel. In between comes a black imprinting foil, and the scale is melted into the surface of the bevel. The die-stamp is compensated in its length to allow heat expansion, thereby assuring accuracy of the length of the scale.


Figure 15: Ancient Embossing Machine for Hot-Foil Printing of Scale

26. Process: Cutting the hairline on the cursor.

A little complex mechanical device is used here. Clamping the cursor on the top, a cam-operated knife moves upward to the underside of the cursor, and cuts a line. Adjusting the cams determines the starting point and end point of the hairline.
The cursor is aligned to ensure a true vertical to the guiding lower edge.
27. Process: Applying black ink into the hairline.

Using a pad of hard felt to apply the ink and a lot of rags to clean up the excess colour. Anyway one gets dirty fingers.
28. Process: Cutting cursor springs.

A small knife fitted to a shoemakers die punch does the job.
29. Process: Bending cursor springs.

A shaped piston of flat steel squeezes the spring down onto a piece of rubber.
The rubber can not escape and pushes the spring to take the shape from the piston.
30. Process: Fitting springs into the cursor.

Good fingernails is a must. Pliers can do it but eventually scratch the cursor.
31. Process: Fitting the body with slides.. Checking the sliding movement.


Figure 16: The two Slides being fitted actually from one End
32. Process: Fitting the cursor on the body. General inspection.

Time for enjoying your product.
33. Process: Packing for shipment.

In this case the customer checks the slide rule when he puts the slide rules in cases of his own.


Figure 17: The Finished Slide Rule

## Reinforced Concrete Slide Rules

## Pierre Vander Meulen



## Introduction

The present article aims to supply an overall survey of the Reinforced Concrete Slide Rules (Rcsr) which where produced during the slide rules era.
In order to better understand the key elements of the survey, this article starts with some reinforced concrete design basics.

## Limitations of this article

The survey intentionally does not deal with concrete slides rules which are not specifically dedicated to the reinforced concrete design.
The following types of slide rules are therefore not considered but could be the ground for another interesting article:

- Concrete mix (composition) design [e.g. Faber-Castell $2 / 62$ or $57 / 62$, Dywidag]
- Water cement ratio determination : [e.g. Charles Brand]
- Concrete Volume Computer [e.g. Unique]


## History

Concrete that includes embedded metal (usually steel) is called reinforced concrete or ferroconcrete.
Reinforced concrete was invented in $\approx 1849$ by Joseph Monier, who received a patent in 1867. J. Monier (18231906) was a Parisian gardener who made garden pots and tubs of concrete reinforced with an iron mesh. J. Monier exhibited his invention at the Paris Exposition of 1867. In 1854 a plasterer, William B. Wilkinson erected a small two-story servant's cottage, reinforcing the concrete floor and roof with iron bars and wire rope, and took out a patent on this type of construction in England. In the 1890s two other Frenchmen, Edmond Coignet and François Hennibique, used reinforced concrete for pipes, aqueducts, bridges, tunnels.
G.A. Wayss with M. Koenen and E. Mörsch made a great contribution to the early development of the theory and practice of reinforced concrete. Since that period many famous names are associated with reinforced concrete such as Kleinlogel, Freyssinet, Magnel, Fuller, ...

## Basics of Rc materials and beam under flexion

The most important basic feature of the reinforced concrete technique is the combination of two materials working together respectively with their best characteristics:

- Concrete : strong in compression and able to be poured
- Steel : strong in tension and able to be bent in order to follow the element shape

The two materials are associated in the same element (beam, slab, column, ..). The reinforcement steel bars (rebars) are kept in place in the concrete mass and perfectly adhere to the concrete (no differential movement). The concrete PH of $+/-12.7$ protects the steel from corrosion (providing the concrete cover is sufficient and the concrete is compact).

It has to be pointed out that concrete and steel have about the same elongation coefficient (12 $10^{-6} /{ }^{\circ} \mathrm{C}$ ). Should it be different, by differential thermal elongation the two materials could loose their adherence and jeopardize the element stability.

The Young modulus or elasticity modulus [E] expresses the ratio between the stress (force divided by section) and the strain (elongation divided by length).


The steel is stiffer than the concrete. The ratio between the two modules $\left(E_{s} / E_{c}\right)$ is symbolized by " $n$ " or " $m$ ". For the same strain in the concrete and the rebar, the stress in the rebar is $n$ times higher than the stress in the concrete. Generally $\mathrm{n}=15$ but other ratios exist (from 8 to 15 , or even 20 to 25 ).

The concrete is a material which is very weak in tension compared to this performance in compression. Nowadays, typically a standard concrete offers an ultimate compressive strength of $400 \mathrm{~kg} / \mathrm{cm} 2[40 \mathrm{~N} / \mathrm{mm} 2]$ while the tensile strength is about a tenth of it.


Nowadays two types of steel (rebars) are used

1. Plain round hot-rolled mild steel bars (dotted line on the diagram below)

- Yield stress $=\quad 220 \mathrm{~N} / \mathrm{mm}^{2}\left[2200 \mathrm{~kg} / \mathrm{cm}^{2}\right]$
- Rupture strength $=\quad 335 \mathrm{~N} / \mathrm{mm}^{2}\left[3350 \mathrm{~kg} / \mathrm{cm}^{2}\right]$
- Allowable stress $\left(\sigma_{\mathrm{e}}\right.$ or $\left.\sigma_{\mathrm{a}}\right)=220 / 1.6 \approx 140 \mathrm{~N} / \mathrm{mm}^{2}\left[1400 \mathrm{~kg} / \mathrm{cm}^{2}\right]$

These bars which were the only one available before appr. 1950 are quasi abandoned. They get a smooth surface (lower adherence) and need hooked ends for the anchorage into the concrete mass.
2. Deformed (high-bond) hot-rolled high-yield steel bars

- Yield stress $=\quad 500 \mathrm{~N} / \mathrm{mm}^{2}\left[5000 \mathrm{~kg} / \mathrm{cm}^{2}\right]$
- Rupture strength $=\quad 580 \mathrm{~N} / \mathrm{mm}^{2}\left[5800 \mathrm{~kg} / \mathrm{cm}^{2}\right]$
- Allowable stress $\left(\sigma_{\mathrm{e}}\right.$ or $\left.\sigma_{\mathrm{a}}\right)=500 / 2.1 \approx 240 \mathrm{~N} / \mathrm{mm}^{2}\left[2400 \mathrm{~kg} / \mathrm{cm}^{2}\right]$





## Why reinforce a beam?

Submitted to a load, the bottom part of a beam is subject to tension and the top part to compression. The concrete having a very poor resistance to tensile forces, a crack takes place, the section is weakened and the beam finally

collapses.

Without the tenor on the beam, this one is just strong enough to support is own weight. But with him the tensile stress is too high and the beam is going to collapse; for the concert, you can just forget it !
If steel bars are added at the bottom, the tensile forces are equilibrated by the steel bars and the top compression forces by the concrete. The beam section is now stable.

## Basic formulas and theory of a section submitted to bending moment

The following hypotheses are made:

- strains in rebars and concrete are the same and proportional to the distance from the neutral axis, at which the strain is zero (Navier's theory)
- plane sections remain plane and perpendicular to the neutral axis after bending (Bernoulli's hypothesis)
- the tensile strength of the concrete is ignored
- there is a stress/strain relationship for both rebars and concrete materials
- in elastic or allowable stress method stress/strain are proportional (Hooke's law) and are the same in tension as in compression
- in ultimate strength design method the concrete and steel stress distribution versus the strain is no longer linear and are considered at failure


## Typical case

A beam is subject to a local external vertical force ( P ). This force generates an internal bending moment (M) which, in turn, puts the top part in compression and the bottom part in tension (inner force F).


## Typical symbols



- $\sigma_{\mathrm{s}}=$ stress in tension reinforcement (tension);
- $\sigma_{\mathrm{c}}=$ maximum stress in concrete (compression);
- $\mathrm{A}_{\mathrm{s}}=$ rebars section;
- $\mathrm{b}=$ section width;
- $\mathrm{E}=$ modulus of elasticity;
- $\mathrm{h}=$ section height (overall height minus the concrete cover);
- $\mathrm{h}_{\mathrm{t}}=$ overall section height;
- $\mathrm{x}=$ neutral axis depth;


## Formulas

The stresses acting on the top part (concrete in compression) are giving the following force: $N_{c}=\frac{1}{2} \sigma_{c} . x . b$ and on the bottom part (rebars in tension with the concrete in tension neglected) $N_{s}=A_{s} \cdot \sigma_{s}$

The horizontal forces balance is giving: $N_{c}=N_{s}$ [first basic equation] or $\frac{1}{2} \sigma_{c} \cdot x \cdot b=A_{s} \cdot \sigma_{s}$
The max stress in concrete is thus: $\sigma_{c}=\frac{2 A_{s} \cdot \sigma_{s}}{b \cdot x}$

Through the Bernoulli's hypothesis we get: $\frac{\varepsilon_{c}}{\varepsilon_{s}}=\frac{x}{(h-x)}$, and through the Hooke's law we get $: \frac{\sigma_{c}}{E_{c}} / \frac{\sigma_{s}}{E_{s}}=\frac{x}{(h-x)}$; we deduce: $x=\frac{E_{s}}{E_{c}} \cdot \frac{\sigma_{c}}{\sigma_{s}}(h-x)$

If we put $\frac{E_{s}}{E_{c}}=n$, we get $x=n \cdot \frac{\sigma_{c}}{\sigma_{s}}(h-x)$ or $x=\frac{n \cdot \sigma_{c}}{\sigma_{s}+n \cdot \sigma_{c}} h$
and if we put $\alpha=\frac{n \cdot \sigma_{c}}{\sigma_{s}+n \cdot \sigma_{c}}$ [3], we get finally $x=\alpha . h \quad$ [4] where $\alpha$ is dependant on the concrete and steel stresses only and where x is the neutral axis depth.

The bending moment is balanced by the normal forces according to: $M=N_{c}\left(h-\frac{x}{3}\right)$ [second basic equation].
This gives through [1] $M=\frac{1}{2} \sigma_{c} \cdot b \cdot x\left(h-\frac{1}{3} x\right)$ or through [4] $M=\frac{1}{2} \sigma_{c} \cdot b \cdot \alpha \cdot h\left(h-\frac{1}{3} \alpha \cdot h\right)$
or $M=\frac{1}{6} \sigma_{c} \cdot b \cdot \alpha \cdot h^{2}(3-\alpha)$
From there $h^{2}=\frac{6}{\sigma_{c} \cdot \alpha \cdot(3-\alpha)} \cdot \frac{M}{b}$; if we put $\beta^{2}=\frac{6}{\sigma_{c} \cdot \alpha \cdot(3-\alpha)}$, then we deduce $h=\beta \cdot \sqrt{\frac{M}{b}} \quad$.[5]
This is the fundamental equation giving the minimum allowable section height for a given moment (M), a given base (b) and the allowable stresses ( $\sigma_{\mathrm{c}}$ and $\sigma_{\mathrm{s}}$ ).

The rebar section is found through [2] and [4]: $A_{s}=\frac{\sigma_{c} \cdot \alpha}{2 \sigma_{s}} b . h$. I we put $\gamma=\frac{\sigma_{c} \cdot \alpha}{2 \sigma_{s}}$, we get through [5] $A_{s}=\gamma \cdot b \cdot \beta \sqrt{\frac{M}{b}}$ and if we put $\gamma \cdot \beta=\delta$ we get finally $A_{s}=\delta \cdot b \sqrt{\frac{M}{b}}$ which gives the rebars section in function of a given moment (M), a given base (b) and the allowable stresses ( $\sigma_{\mathrm{c}}$ and $\sigma_{\mathrm{s}}$ ).

## Practical conclusion

From what is explained above, from the 7 parameters $h, b, M, A_{s}, \sigma_{c}, \sigma_{s}, n$ and the 2 basic equations, we have to fix 5 parameters to be able to solve the equation system.
A typical situation is to define the height of a section (h) and the amount of rebars $\left(\mathrm{A}_{\mathrm{s}}\right)$, having selected the base of the section (b), the materials characteristics ( $\sigma_{\mathrm{c}}$ and $\sigma_{\mathrm{s}}$ ) and the value of n .

## Practical example

From one of the picture above, we consider the following known parameters:

- $\mathrm{n}=15$
- $\mathrm{b}=20 \mathrm{~cm}$
- $\mathrm{M}=100 * 590 / 4+138 / 100 * 590 * 590 / 8=74794 \mathrm{kgcm}$
- $\sigma_{\mathrm{c}}=60 \mathrm{~kg} / \mathrm{cm}^{2}$ (rather low nowadays)
- $\sigma_{\mathrm{s}}=1200 \mathrm{~kg} / \mathrm{cm}^{2}$ (rather low nowadays)

What is the minimum allowable height and the required rebars section?
Through $\alpha=\frac{n \cdot \sigma_{c}}{\sigma_{s}+n \cdot \sigma_{c}}$ we find $\alpha=0.429$
Through $\beta^{2}=\frac{6}{\sigma_{c} \cdot \alpha .(3-\alpha)}$ we find $\beta^{2}=0.090$ or $\beta=0.301$
Through $h=\beta \cdot \sqrt{\frac{M}{b}}$ we find $\mathrm{h}=18.4 \mathrm{~cm}$ and therefore (with a concrete cover of 3 cm ) ht=21.4 cm.
Through $\gamma=\frac{\sigma_{c} \cdot \alpha}{2 \sigma_{s}}=0.0107$ and $\gamma . \beta=\delta=0.0032$, and $A_{s}=\delta . b \sqrt{\frac{M}{b}}$ we find $\mathrm{A}_{\mathrm{s}}=3.95 \mathrm{~cm}^{2}$
and trough $x=\alpha . h, \mathrm{x}=7.9 \mathrm{~cm}$.

## Conclusion

The overall height of 30 cm is sufficient (> minimum of 22 cm ) and 2 rebars of $16 \mathrm{~mm}\left(4.02 \mathrm{~cm}^{2}\right)$ are adequate.

## Typical calculation examples on RCSL

We take, for the following examples, the data selected in the practical example of the above paragraph.

## On a Nestler 43, system Hoffmann


known variables:

- $[\mathbf{M}]=74794 \mathrm{kgcm}$ : on the scale A
- $[\mathbf{b}]=20 \mathrm{~cm}$ : on the scale B to be put in front of $[\mathbf{M}]$ on scale A
- $\left[\sigma_{\mathrm{s}}\right]=1200 \mathrm{~kg} / \mathrm{cm}^{2}$ : by selecting the slide marked $\sigma_{\mathrm{e}}=1200 \mathrm{~kg} / \mathrm{cm}^{2}$
- $\left[\sigma_{c}\right]=60 \mathrm{~kg} / \mathrm{cm}^{2}$ : on the specific scales marked $\mathrm{h}, \mathrm{X}$ and $\mathrm{f}_{\mathrm{e}}$
unknown variables to be found:
- $\quad[\mathbf{h}]=18.4 \mathrm{~cm}$ is found on the scale D under the selected $\left[\sigma_{\mathbf{c}}\right]$ on scale $h$
- $\quad[\mathbf{x}]=7.9 \mathrm{~cm}$ is found on the scale D under the selected $\left[\sigma_{\mathbf{c}}\right]$ on scale X (after displacement of the slide to get the 1 on the previous place of the 10)
- $\quad\left[\mathbf{A}_{\mathbf{s}}\right]=3.95 \mathrm{~cm}^{2}$ is obtained by multiplying 0.1975 found on the scale $D$ under the selected $\left[\boldsymbol{\sigma}_{\mathbf{c}}\right]$ on scale $\mathrm{f}_{\mathrm{e}}$, by $[\mathbf{b}]$



## On an Aristo 939, sytem Göttsch


known variables:

- $[\mathbf{M}]=747.94 \mathrm{kgm}:$ on the scale A
- $[\mathbf{b}]=20 \mathrm{~cm}$ : on the scale B to be put in front of $[\mathbf{M}]$ on scale A
- $\left[\sigma_{\mathrm{s}}\right]=1200 \mathrm{~kg} / \mathrm{cm}^{2}$ : by selecting the slanted mark 1200 on the cursor
- $\left[\sigma_{\mathrm{c}}\right]=60 \mathrm{~kg} / \mathrm{cm}^{2}$ : on the specific scale marked $\sigma_{\mathrm{b}}$
unknown variables to be found after bringing the slanted cursor mark in coincidence with the 60 on the specific scale $\sigma_{b}$ :
- $[\mathbf{h}]=18.4 \mathrm{~cm}$ is found on the scale D at the main cursor hairline
- $\quad[\mathbf{K}]=0.935$ is found under the vertical cursor mark 1200 on the $K$ scale
- $\left[\mathbf{A}_{\mathbf{s}}\right]=3.95 \mathrm{~cm}^{2}$ is obtained, through a normal operation on scale D by dividing $[\mathbf{h}]$ by $[\mathbf{K}]$ and by multiplying it by [b]


On a Faber-Castell 371, system Torda or on a F-C 3/11

known variables:

- $[\mathbf{M}] /[\mathbf{b}]=747.94 / 0.20=3739 \mathrm{~kg}$ : on the scale A
- $\left[\sigma_{\mathbf{c}}\right]=60 \mathrm{~kg} / \mathrm{cm}^{2}$ selected on the scale " $\sigma b$ is to be located just under 3739 on the scale A
- $[\mathbf{v}]$ is intermediately calculated : $[\mathbf{v}]=\left[\boldsymbol{\sigma}_{\mathbf{s}}\right] /\left[\boldsymbol{\sigma}_{\mathbf{c}}\right]=1200 / 60=20$
unknown variables to be found:
- $\quad[\mathbf{h}]=18.4 \mathrm{~cm}$ is found on the scale D just under $[\mathbf{v}]$ located on the scale $h$
- $\left[\mathbf{A}_{\mathbf{s}}\right]=3.95 \mathrm{~cm}^{2}$ is obtained by multiplying 0.1975 found on the scale $D$ under the selected $[\mathbf{v}]$ on scale $f_{e}$, by $[\mathbf{b}]$
- $[\mathbf{x}]=7.9 \mathrm{~cm}$ is found on the scale D just under $[\mathbf{v}]$ located on the scale x (after displacement of the slide to get the 1 on the previous place of the 100 )



## Survey of Rcsr

## Methodology

In the author's knowledge, virtually no survey of this kind has been produced up to now.
The method used is a twofold one:

- Physical gathering of RCSR in the collection of the author and others, and deep examination of each specimen (with possibly the user's manual if available)
- Documents survey (catalogues, articles, Internet, ...)

The result of this survey is summarized in a table and is, for sure, far from exhaustive (see Appendix 1).
The empty cells are resulting either from a non relevant characteristic or from non available information. A synthesis of the survey is firstly given, highlighting the most significant conclusions.

## Survey most significant figures

From the survey given in appendix 1, the following main figures could be highlighted:

- Number of inventoried specimens: 45
- for wire-mesh : 4
- for concrete floors: 1
- for reinforced concrete beams (and slabs): 40
- for allowable stress method : 37
- for ultimate limit state method: 3
- Number of physically examined specimens: 21
- Fabrication year : from 1913 to 1975
- Number of specific scales : 3 to 18 ( 37 for the specialized Rcsr for wire mesh selections)
- Number of duplex slide rules: 7


## Survey most significant conclusions

The column () refers to the numbers located at the top of the table as shown in Appendix 1.

## Column (5) : system

Many systems exist. The differences are mainly due to a more and more sophisticated approach to find the intermediated values $\alpha, \beta, \gamma, \delta \ldots$ Furthermore, more sophisticated cursors, together with specific scales, where developed.

## Column (9) : allowable steel stress $\sigma_{a}$

The steel stress is either fixed by the used scales or left to the free choice of the user (integrated in an intermediate parameter to be calculated or to be selected on a two cycles scale).

The range is rather extended : from $800 \mathrm{~kg} / \mathrm{cm}^{2}\left[80 \mathrm{~N} / \mathrm{mm}^{2}\right]$ up to $3500 \mathrm{~kg} / \mathrm{cm}^{2}\left[350 \mathrm{~N} / \mathrm{mm}^{2}\right]$. Nowadays, the lower values are no longer used; a standard value is $2400 \mathrm{~kg} / \mathrm{cm}^{2}$.

## Column (10) : allowable concrete stress $\sigma_{b}$

The concrete stress is either fixed by the used scales or left to the free choice of the user (integrated in an intermediate parameter to be calculated).

The range is rather extended : from $10 \mathrm{~kg} / \mathrm{cm}^{2}\left[1 \mathrm{~N} / \mathrm{mm}^{2}\right]$ up to $180 \mathrm{~kg} / \mathrm{cm}^{2}$ [ $\left.18 \mathrm{~N} / \mathrm{mm}^{2}\right]$. Nowadays, the lower values are no longer used; a standard value is $100 \mathrm{~kg} / \mathrm{cm}^{2}$.

## Column (11) : Elasticity modules ratio n

Most of the RCSR are calibrated for $\mathrm{n}=15$. This value has been adopted by most of the national design Codes and Standards. Nevertheless some other values were imposed (from 8 to 15 ).

If n is fixed by the Rcsr , the passage to another n is nevertheless possible by a preliminary proportional modification of steel allowable stress $\left(\sigma_{\mathrm{a}}{ }^{*} 15 / \mathrm{n}\right)$ and, at the end of the calculation, by a proportional multiplication of the found steel section (As*15/n); this is explained by the equation [3].

## Column (12) : Number of scales

From the conclusion, it appears that in addition to the E moduli ration, 4 known variables are to be fixed in order to solve the standard bending moment problem. M and b are generally put on the A and B scales and the two remaining variables are to be found through specific scales or by conventional calculation.

In order to help the designer, some specific scales where developed on the Rcsr. The sophistication and number of these specific scales are characteristic of the different systems.

The Aristo 939 and the Faber-Castell 2/31 are equipped with sophisticated cursors allowing the selection of many allowable stresses and possibly several n. These two Rcsr seem to be the most developed models ever produced before the Aristo 940 (dedicated to the limit state design).

## Column (12) : Standard scales

$M$ and $b$ are generally put on the $A$ and $B$ scales and the two remaining variables are to be found through specific scales on the D scale, either by direct $\beta$ calc or through specific scales; this is due to the particular structure of the equation [5].


Furthermore two of the duplex slide rules are showing on one face a full conventional system of scales (FaberCastell $2 / 31$ and Nestler 0440)

## Column (14) : Design method

Most of the RCSR are built in order to allow for the so called allowable or elastic stress method. This method extensively used during more than 60 years has been progressively superseded by the so called "limit state design method". Only three among the 45 Rcsr of the survey are built for that particular design method. It is understandable, taking into account the introduction of this method around the year 1962, and the time needed for its international diffusion (close to the end of the slide rules era), that only few specimens are dedicated to it.

## Column (15) : Backside data

As foreseen, most of the non-duplex Rcsr are showing at the backend some tabulation allowing the designer to transform the calculated steel section in an equivalent number of rebars. Sometimes a short description of the slide rule use is also given.

## APPENDIX



REINFORCED CONCRETE SLIDE RULES SURVEY
INVENTORY AND MAIN CHARACTERISTICS
(See table next pages)

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |  | (12) |  |  | (14) | (15) | (16) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 를 |  | R | $\begin{aligned} & \text { N } \\ & \text { d } \end{aligned}$ |  |  |  |  | $\stackrel{\sim}{6}$ | E | $\begin{aligned} & \mathscr{N} \\ & \frac{\mathscr{E}}{6} \end{aligned}$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | allowable <br> concrete stress <br> [kg/cm2] (or concrete type) |  |  |  |  | $\begin{aligned} & \underset{E}{\theta} \\ & \stackrel{\rightharpoonup}{0} \\ & \end{aligned}$ |  |  |  |
|  |  |  |  |  |  |  |  | $\mathrm{kg} / \mathrm{cm} 2$ |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Alro |  | yes | $\begin{gathered} \hline \text { BO. B. } \\ \text { (note 5) } \end{gathered}$ |  |  | Diam 13 |  | 1200 | 40 to 60 |  | 7 | A/D//C/B | alu |  | A | rebars sections data | Disc |
| Aristo | 40140 |  |  | Baustahl gewebe | For wire mesh only | 20 |  | $\begin{gathered} \hline \text { BSt 50/55 } \\ \text { (note 2) } \\ \hline \end{gathered}$ | Bn 250 |  | 11 | A//B/CI/C//D | plastic |  | A | wiremeshes sections data |  |
|  | 80136 |  | $\begin{gathered} \text { Bemessungs } \\ \text { schieber } \\ \hline \end{gathered}$ | Baustahl gewebe | For wire mesh only | 12,5 | +/-1965 | 2400 to 2800 | 30 to 120 |  | 10 | A//B/Cl/C//D | plastic |  | A | $\begin{array}{\|c} \hline \text { techical tables + } \\ \text { formula } \\ \hline \end{array}$ |  |
|  | 90184 | yes | Bemessungs schieber | Baustahl gewebe | For wire mesh only | 25 | 1964 | 2400 / 2800 | 50 to 90 |  | 37 | A//B/CI/C//D | plastic | yes | A |  |  |
|  | 90184N |  | Bemessungs schieber | Baustahl gewebe | For wire mesh only | 25 | 1964 |  |  |  |  |  | plastic | yes | A |  |  |
|  | 938 |  |  |  |  | 25 |  |  |  |  |  |  |  |  |  |  |  |
|  | 939 |  | Stahlbeton | Göttsch |  | 25 |  | 800 to 2000 | 30 to 120 | 8 | 8 | A/B/C//D | plastic | yes | A | instructions + | the front scales |
|  |  |  |  |  |  |  |  | 800 to 2200 |  | 10 |  |  |  |  |  | rebars sections | are reproduced |
|  |  | yes |  |  |  |  | 1969 | 800 to 3500 |  | 10 |  |  |  |  |  |  | very re |
|  |  |  |  |  |  |  |  | 1200 to 2600 |  | 15 |  |  |  |  |  |  | scale, and are |
|  |  | yes |  |  |  |  | 1968 | 1200 to 2800 |  | 15 |  |  |  |  |  |  | used for an |
|  |  | yes |  |  |  |  | 1974 | 1200 to 3500 |  | 15 |  |  |  |  |  |  | approximate calculation + calc of "x" and se |
|  | 940 | yes | Stahlbeton n-los (DIN 1045) | Göttsch |  | 25 | 1972 | $\begin{gathered} \hline \text { BSt } 22 \text { to } \\ \text { BSt50 } \\ \text { (note 2) } \end{gathered}$ | $\begin{array}{\|c\|} \hline \text { Bn150/250/ } \\ 350 / 450 / 550 \\ \text { (note1) } \\ \hline \end{array}$ | NA | 18 | A//B/BI/CI/C//D | plastic | yes | U | instructions + rebars and mesh sections data |  |
| $\begin{gathered} \text { Dennert } \\ \& \end{gathered}$ | 38/26 |  |  |  |  | 30 | <1935 |  |  |  |  |  |  |  |  |  |  |
| Pape | 38/28 |  |  | Lewe |  | 25 | 1913 |  |  |  |  |  |  |  |  |  |  |




## Notes in the tables above:

$\left.\begin{array}{|l|c|c|}\hline \text { notes } & 1 & \begin{array}{c}\text { Bx induces a of } \begin{array}{c}\mathrm{Bx} / 3 \text { (depending of the steel } \\
\text { and construction type) }\end{array} \\
\hline\end{array} 2^{2} \\
\hline & \begin{array}{c}\text { St or BSt induces a a of St/1.75 (depending of } \\
\text { the steel and construction type) }\end{array} \\
\hline & 4 & \text { one stress per slide side }\end{array} \right\rvert\, \begin{array}{c}\text { exact year if inferred from an examined model } \\
\hline\end{array}$ NA \(\left.\begin{array}{c}The name appears on a elementary typewriter <br>

typed user's manual\end{array}\right]\)| Not applicable |
| :---: |

## Long-Scale Slide Rules Revisited

## Edwin J. Chamberlain

## Introduction

The quest for greater calculating precision with slide rules started at the beginning of slide rule history. This pursuit has resulted in the development of many different forms of "Long-scale Slide Rules" with extended scale lengths. This report is an update of a paper on long-scale slide rules published by the Oughtred Society [11]. It reviews the historical development of long-scale slide rules, defines the different categories and formats, and presents many examples - including some new examples that have come to my attention since the OS paper was published.

The "long-scales" that I refer to in this paper are the single cycle calculating scales (sometime broken into segments) used for multiplication and division. "Long-scale slide rules" are those slide rules with calculating scale lengths greater than the lengths of the C and D scales on common $25-\mathrm{cm}$ slide rules. For time and space considerations, relatively common $50-\mathrm{cm}$ slide rules are not included in this report. They may be the subjects of another paper. However, for a sense of completeness, slide rules that provide relatively long-scales in a small format, such as Fowler long-scale pocket watch style circular slide rules and the Nestler Präzision $25-\mathrm{cm}$ slide rule with segmented $50-\mathrm{cm}$ long-scales on a $25-\mathrm{cm}$ scale body will be included, even though their scale lengths do not exceed $50-\mathrm{cm}$.

In my JOS [11] article, I also included an extensive discussion of the precision of long-scale slide rule calculations. I will forego repeating that discussion here other than to say that the precision of the longest scale slide rule known to be commercially made - the Loga 24-meter Rechenwalze - is 4 to 5 digits with interpolation, while the precision of the common $25-\mathrm{cm}$ long-scale slide rule is 2 to 3 digits. A plot of precision versus scale length showed that the precision of calculations with slide rules increases about one digit for every order of magnitude increase in length of the calculating scale. The plot also showed that the precision is greater at the beginning of the scale than at the end of the scale, and the precision of spiral slide rules is more nearly the same at both ends.

As we will see, long-scale slide rules have taken several different forms, including: 1) linear or straight; 2) circular; 3) cylindrical and 4) continuous ribbon or tape. There are also variants on these formats including multi-segmented, spiral, concentric circle, helix and saw tooth scales. The following presents a summary of the historical development of long-scale slide rules, and their formats and scale lengths. The slide rules discussed are tabulated in Tables I and II.

## History of Long-Scale Slide Rules - The Early Days of Oughtred and Delamain

The first known 'long-scale' slide rule may have been the circular slide rule made for William Oughtred by Elias Allen, an instrument maker, in about 1632. This slide rule was featured on the front and back covers of the Journal of the Oughtred Society [29] in March of 1996. It is possibly the oldest slide rule known to exist. An original made by Elias Allen is in the Whipple Museum in Cambridge, England. The disk is about $32-\mathrm{cm}$ diameter, and the number calculating scale has a length of about $76-\mathrm{cm}$. That makes the number scale on this slide rule about three times longer than the number scale on the common $25-\mathrm{cm}$ slide rule of the $20^{\text {th }}$ century. According to Cajori [10], Oughtred may have designed this slide rule as early as 1621 , shortly after making a slide rule of sorts by placing two rulers with double lines of Gunter's numbers adjacent to each other.

Prior to that, it appears that there were Gunter rules with double lines of numbers as long $1.8-\mathrm{m}$. Cajori [10] reported that in 1632 William Forster, a student of Oughtred, told him that he had been making " 6 -ft long" Gunter rules. Gunter rules were relatively common for men of science in England at that time. One cycle of the two cycle calculating scale on Forster's Gunter rule would have had a length of about $90-\mathrm{cm}$. Oughtred responded to Forster by saying that it (Forster's six foot long Gunter scale) "was a poore invention" with a "trouble some performance". He went on to describe a slide rule of sorts made of ". . two Rulers . . . to be used by applying one to the other. . .". He also showed Forster ". . .those lines cast into a circle or Ring, with another moveable circle upon it." Oughtred was describing a two-disk circular slide rule. He had also developed the idea of a circular slide rule with a spiral scale
years before this conversation with Forster. Forster wondered why Oughtred did not mention his ideas earlier, when "he had bin so liberall . . . in other parts of Art [mathematics and science]" to his students. Oughtred responded that he believed strongly that the Artist [e.g., the student] should be "well instructed in the sciences" before succumbing to instruments as "doers of tricks". Oughtred simply believed that it was more important for his students to be well grounded in the mathematics of the day than to be proficient at making calculations that were normally left for a technician to do. As we will see, this philosophy left him vulnerable to another of his students.

Forster respected Oughtred's views. However, another student at about the same time, Richard Delamain, apparently did not. Once Delamain found out about Oughtred's circular slide rules, he got busy, borrowing some ideas from Oughtred and developing some of his own. He began making and writing about circular slide rules. The story has been told by Cajori [10]. Delamain favored a two-disk configuration with a single indicator, while Oughtred preferred a single disk with a pair of indicators. The first (known) circular slide rule attributed to Oughtred was of the single disk type - the Elias Allen example discussed above. In his pamphlet, Grammelogia IV, published in 1632, Delamain showed the layout for a two-disk calculating device (Figure.1) each disk with a pair of calculating scales laid out on five concentric circles. The scale length on the outer "fixed" disk was about $140-\mathrm{cm}$ and was about $112-\mathrm{cm}$ on the inner movable disk.

Delamain's creativity may have been that he conceived of the "long-scale" slide rule in concentric ring format in difference to the spiral scale format that Oughtred had told him about, and that he promoted the double disk over the single disk format. However, one thing, that Delamain apparently did not recognize, is that by running his outer


Figure 1: Delamain spiral circular slide rule scale inwards, he lost a big advantage of the spiral and concentric ring slide rule formats. That advantage being that, for such scales running outwards on a disk, the precision is improved at the higher end of the scale because the gradation spacings increase as the diameter of the rings (or revolutions) increase.

Moreover, Delamain was really thinking big about "long-scale" slide rules. In addition to his concentric ring slide rule, he also conceived of a "Great Cylinder" slide rule a "yard" in diameter with 10 or more pairs of ganged fixed and moveable calculating disks. It is doubtful that such a device was made, however the scale length could have exceeded $30-\mathrm{m}$. Delamain could be thought as the father of the cylindrical slide rule, as that idea apparently did not come from Oughtred.

## History of Long-Scale Slide Rules $-17^{\text {th }}$ through the $19{ }^{\text {th }}$ Centuries

Most of the innovations in long-scale slide rule technology came in the period between Oughtred's 'invention' of the circular slide rule in about 1621 , and the advent of the modern age of slide rules in the $20^{\text {th }}$ century. My source for much of this early history was the reprint of Florian Cajori's [9] "A History of the Logarithmic Slide Rule". The different long-scale slide rules found in this study are listed in Tables I and II.

## The $17^{\text {th }}$ Century

Most of the early developments in slide rule technology were made in England. As we have seen, 'long-scale' slide rules were amongst the first of the innovations that appeared. In addition to the work of Delamain, Cajori reported
that Milburne of Yorkshire designed a spiral form of slide rule in about 1650. Cajori also found that about this same time that John Brown projected Gunter's line into a kind of spiral of 5, 10, and 20 turns."

Remarkably, the Science Museum in London has on display two very early long-scale spiral slide rules dating to the mid 1600s, including an original by John Brown [56] that dates to about 1660. The other is a copy of a spiral slide rule designed by William Oughtred, signed Henr Sutton fecit, and dated 1663 [57].

The John Brown spiral slide rule is made on a wooden disk, about $15-\mathrm{cm}$ diameter. This disk has three spiral scales: 1) the innermost one a 7 -revolution tangent scale; 2) the outermost scale being a 5 -revolution number scale; and 3) a 5-revolution sine scale in between these two scales. The scales all wind their ways outwards on the disk. A pair of brass indicator arms facilitate the calculations. The number scale is about $213-\mathrm{cm}$ long. With care, the John Brown spiral slide rule can be read to about four digits. It should be mentioned that John Brown was probably mentored in the art of making spiral slide rules by his father, Thomas, who is credited by Hopp [26] for making the first spiral slide rule in 1631. Hopp suggests that William Milburne may have been the inventor of the spiral slide rule, but as we have seen earlier, Oughtred has claimed credit for that invention.

The Sutton spiral slide rule is laid out on a brass disk about $34-\mathrm{cm}$ diameter. The scale layout is most curious. It starts near the center of the disk with four windings of a pair of scales for calculating functions of angles. The number scale starts at the beginning of the fifth winding and continues for five additional windings, while the angle scales continue for an additional two windings. A pair of brass indicator arms facilitate the calculations. The fiverevolution scale has a length of about $280-\mathrm{cm}$. Like its John Brown contemporary, the Sutton spiral slide rule can also be read with interpolation to about four digits.

Another finding of Cajori [9] was a slide rule made in England in the mid 1600s by Horner with a calculating scale made up of many parallel segments. This slide rule appears to be the forerunner of the gridiron slide rule that was developed later in the $19^{\text {th }}$ century. Cajori did not report any details, so we do not know what scale lengths were obtained. He also mentions a semicircular slide rule conceived by a German writer named Biler in 1696. It is described as having sliding concentric semicircles, but no details are given.

## The $\mathbf{1 8}^{\text {th }}$ Century

The idea of the long-scale slide rule was also promoted by John Ward in 1707. Ward found that pocket slide rules having length of "nine inches or a foot long . . at best do but help guess at the Truth." He recommended slide rules of "two or three feet" ( 60 to $90-\mathrm{cm}$ ) length to get the accuracy required for gauging liquid spirits in casks. One must recognize that a part of the problem that Ward and the gauging profession faced was that the calculating scales on ordinary slide rules were often crudely laid out. It was apparently easier in his time to more accurately layout the scales on long slide rules.

In 1733, Benjamin Scott described a circular slide rule nearly $46-\mathrm{cm}$ diameter with a circular scale having a circumference of about $1.5-\mathrm{m}$. According to Cajori, Scott was unaware of any forerunners of his work. A few years later in 1748, George Adams, an instrument maker in England, engraved a spiral scale with 10 windings on a brass plate about $30-\mathrm{cm}$ diameter. Although it is not known for sure, the scale on Adams' slide rule may have been as long as $5-\mathrm{m}$.

In the 1700 's, there were also some developments in long-scale slide rule technology in other European countries. In 1717 in Italy, for instance, Bernardus Facini designed a spiral scale slide rule that had a scale length of about 120cm . Interpolation of readings was aided by the inclusion of vernier-like markings that run on a band just outside the spiraling scale. The only known copy of Facini's slide rule is in the Adler Planetarium \& Astronomy Museum in Chicago. A photo of this disk appeared on the cover of the Journal of the Oughtred Society [31].

In Germany in the 1770 's, the scientist Johann Heinrich Lambert, had linear slide rules made that had scale lengths of "4-feet" (about 130-cm). In England (c1775) the Robertson [45] sliding Gunter slide rule employed a calculating scale about $72-\mathrm{cm}$ in length. Examples of this slide rule are known to be in two private collections.

In 1727 in France, Jean Baptiste Clairaut described a circular slide rule having a $53-\mathrm{cm}$ diameter, which had a large number of concentric circles, one of which was a long-scale number scale. While not known, the length of this calculating scale could have been greater than $1.5-\mathrm{m}$. Even more impressive is another long-scale slide rule of

Clairaut. In 1716, he designed a multi-segment linear slide rule laid out on a square of one foot, filled with parallel lines making up a single number calculating scale. Cajori [9] reported that this slide rule had a scale length of " 1500 French feet". Given about $32.5-\mathrm{cm}$ per one old "French pied" (foot), the length would be an unlikely $500-\mathrm{m}$. Perhaps something was lost in the translation. This slide rule appears to be one of the first of the gridiron type, but its details remain a mystery.

Later in the $18^{\text {th }}$ century, the Englishmen William Nicholson made several contributions to increasing the precision of slide rule calculations. In 1787, Nicholson described a straight slide rule having a double line of numbers (2-log cycles) $610-\mathrm{cm}$ long. According to his design, the scale was broken down into 10 segments. This slide rule body must have been about $60-\mathrm{cm}$ long. Nicholson even devised a kind of runner to help with the calculations.

Nicholson was the first to design a gridiron slide rule. Gridiron slide rules break the calculating scale into a series of segments laid out one below the other in parallel. There is no slide in the conventional sense. Nicholson's gridiron slide rule had 10 -segments (each successive segment repeating the last half of the previous segment) and a total scale length of over $3-\mathrm{m}$. A beam compass-like device that slides over the surface of the rule facilitates calculations. It was described in the third edition (1798) of the Encyclopedia Britannica [17] and an illustration shown in my earlier long-scale paper [11].

Nicholson also developed a circular slide rule having a single line of numbers made up of three concentric circles,
 illustrations of Nicholson's slide rules taken from his writings, but stated that it is uncertain if any of his slide rules were constructed and sold.

## The $1{ }^{\text {th }}$ Century

The $19^{\text {th }}$ century was characterized by great advances in adopting and improving slide rule technology. The industrial revolution, accompanied by expanding needs of science, technology and commerce for efficient and accurate means of making calculations, caused a great rise in slide


Figure 2: Tavernier-Gravet long-scale slide rule rule technology. This was a period of maturity for long-scale slide rule technology.

The reader will recall that the first long-scale calculating rules based on the logarithmic scale were not slide rules, but were Gunter rules. In the early 1600s William Forster's made Gunter rules with two cycle calculating scales having a single cycle length of about $91-\mathrm{cm}$. However, most Gunter rules made in later years have " $2-\mathrm{ft}$ " long 2cycle number scales, so the scale length for a single cycle ( $30.5-\mathrm{cm}$ ) was not much longer than on the common 25cm slide rule. However, a modification of the " $2-\mathrm{ft}$ " Gunter rule in my collection, made in the mid 1800 s , called the Donn Navigation Rule, carried a single cycle scale having a length of about $61-\mathrm{cm}$.

The French were also busy developing long-scale slide rules in the mid to late 1800s. Cajori [10] reported that Delamoriniere and Delaveleye made linear slide rules with scale lengths ranging from $50-$ to $115-\mathrm{cm}$ (c.1863), and long-scale bi-segmented slide rules were designed by Mannheim (c1850s), and by E. Péraux, named the "Échelle Logarithmique" (c1860s). Unfortunately, few details of these slide rules were reported.

However, we do have complete details of two different long-scale slide rules, based on bi-segmented scales, made by the well-known French slide rule maker, Tavernier-Gravet. One described by von Jezierski [68 \& 69] is a $100-\mathrm{cm}$ long-scale on $50-\mathrm{cm}$ slide rule body (with one slide) designed by Lallemand (c1875). This slide rule broke the calculating scale into two segments, the scale pair at the top margin of the slide running from 1 to 3.16 on the upper stator and 3.16 to 10 on the upper edge of the slide, and on the bottom margin from 1 to 3.16 on the lower edge of the slide and 3.16 to 10 on the top edge of the lower stator.

Another of the Tavernier-Gravet long-scale slide rules with bi-segmented scales from a private collection is shown (Figure. 2) on Ron Manley's slide rule web site [39]. This example has two slides and has $50-\mathrm{cm}$ long-scale sets on a $25-\mathrm{cm}$ slide rule body. It has two different sets of long-scales. The scale pair at the top margin of the top slide running from 1 to 3.16 on the upper stator and 1 to 3.16 on the upper edge of the slide, and from 3.16 to 10 on the lower edge of the slide and 3.16 to 10 on the edge of the stator. The scale pairs on the lower slide are identical to those on the Lallemand T-G slide rule. The precision of calculations with the Lallemand $100-\mathrm{cm}$ version may be a little better than on the $50-\mathrm{cm}$ version, but the $50-\mathrm{cm}$ version is a little more efficient because the two slides provide the choice of using matched or folded scales. Perhaps the two-slide version of the T-G long-scale slide rules was designed by Mannheim, for it is well known that Tavernier-Gravet made slide rules with Mannheim's designs.

It was in the 1800's before any important developments in long-scale slide rule technology occurred in the United States. It appears that the first long-scale slide rule made commercially in the United States was the $20-\mathrm{cm}$ diameter circular slide rule designed Aaron Palmer in the 1840 's. John E. Fuller improved this slide rule with the addition of a Time Telegraph scale on the reverse and copyrighted in 1846. Details of this slide rule have been reported by Feazel [18]. It sold under the Fuller-Palmer name in fairly large numbers over the next 20 years. A copy in my collection has a calculating scale of about $67-\mathrm{cm}$ in length.

The Nystrom "Calculator" (circular slide rule) appeared in the US in 1851, shortly after the Fuller-Palmer. The Nystrom [40] is elegantly engraved on a $24-\mathrm{cm}$ diameter brass disk - somewhat a reminder of the early Oughtred circular slide rule. A kind of vernier scale is built into the slide rule and cursor markings to aid in the interpolation between gradations. It appears that, with the vernier, readings can be resolved to four digits at both ends of the scale. Unfortunately, I did not have a copy of this slide rule to examine closely. The Nystrom Calculator is very rare, and highly desired for collections. One was sold at the Skinner Science \& Technology [61] auction in 1997 for $\$ 10,350$.

In the mid 1800s, Ferdinand R. Hassler developed a $68-\mathrm{cm}$ long slide rule for his personal use at the US Coast and Geodetic Survey. This slide rule was arranged so that two broad wood rules could slide adjacent to each other. Brass end posts on opposite ends of each piece and a tongue and groove arrangement kept the sliding pieces adjacent to each other. There must have also been a sliding indicator, but it is missing from the example studied. On one side, there is a pair of 10 -segment calculating scales, one scale set on each of the wood strips. One long-scale set runs from 1 to 100 and the other runs folded from $8.92 \times 10^{2}$ to $8.92 \times 10^{4}$. The length of a single cycle in this scale pair is about $3-\mathrm{m}$. This slide rule could make calculations to about four digits precision. The other side of the Hassler slide rule had 10segment scale sets for 2- and 3cycles. These scale sets ran from $1 \times 10^{6}$ to $1 \times 10^{8}$ and $1 \times$ $10^{6}$ to $1 \times 10^{9}$. The Hassler Geodetic slide rule was obviously a special purpose long-scale slide rule. The only example known is shown on the Internet site of the National Institute of Standards and Technology Virtual Museum [41]. It may be one of a kind.

Clairaut's and Nicholson's $17^{\text {th }}$ century ideas about gridiron scales were followed up on in the $1{ }^{\text {th }}$ century by Everett, Hannyngton, Cherry, Billeter,


Figure 3: Hannyington's gridiron slide rule Scherer, Evans and Proell, but not until the later half of the $19^{\text {th }}$ century. The most successful (in terms of examples making it into collections) was designed by Hannyngton. Hannyngton's gridiron slide rules (1880s - 1920s) broke the scale into segments on
parallel boxwood square rods [14]. Each rod on the base set of scales repeated half of the scale segment on the previous rod. The sliding set of rods is ganged and nests into the spaces between the base set of rods so that the scale markings are adjacent to each other. The sliding set of rods is one half the length of the base set. This arrangement keeps the slide from falling off one end of the base when making calculations. Aston \& Mander, the English maker, made at least two versions of the Hannyngton, one with 5 rods per scale and a long-scale length of $159-\mathrm{cm}$ and the other with 10 rods per scale and a scale length of $318-\mathrm{cm}$. These are sometimes referred to as Hannyngton's "small (60-in)" and "large (120-in)" "extended" gridiron slide rules. An example is shown in figure 3.

Cherry's gridiron slide rule (1880) took a little different form. According to Pickworth [47], the Cherry Calculator had segmented scales laid out in a serious of parallel lines on the base. The lines were laid out on a slight bias (Figure. 4) so that the end of a segment line is at the same elevation as the beginning of the next segment line. If the lines were laid out on sheet of paper and wrapped around a cylinder of appropriate diameter, a helix scale much like one on the Fuller cylindrical slide rule would be formed. A matching set of lines was laid out on a transparent sheet that functioned as a slide. Each of the four corners of the base and transparent slide scale sets has index marks. These index marks and the biased scale segments facilitate the calculations. The advantage of this arrangement is that there is no need to repeat parts of scale segments.

Towards the end of the $19^{\text {th }}$ century, the Swiss company Billeter (c1890) also made gridiron slide rules - unlike like the Cherry model - that had the scale segments in parallel lines running straight across the tablet. The base scales were repeated. The glass slide had paper scale strips glued to the underside of the glass. According to Joss [34], the Billeter Rechentafel came in four sizes: $0.5-, 1-, 4-$ and $8-\mathrm{m}$ in scale length, with the scale broken into 4,8 and 10


Figure 4: Cherry's gridiron slide rule
segments for the first three sizes. The details are not known for the largest 8 -m model.
Printed graphic tables are another class of long-scale logarithmic calculating devices. They are similar to gridiron slide rules, but do not have sliding pieces. Cajori [9] reported that in 1846 Léon Lalanne designed a Tableau Graphique, which may have been the first graphic table of calculating line segments. However, Cajori does not give sufficient detail to be sure. Later in the late 1800s, Loewe [38] published his Rechenscalen für numerisches und graphisches Rechnen in Germany. The segments run vertically in Loewe's graphic calculating table in 50 segments, on five pages, for a total long-scale length of about $10-\mathrm{m}$. A pair of dividers is used to facilitate the calculations. Details were reported by Holland [24]. Holland also reported that Anton Tichy [65] published the book "Graphische Logarithmen-Taflen" in Austria in 1897, but gave no details.

The Science Museum in London has a third spiral slide rule in its collection, one made by Dixon in 1882. Dixon's Combined Circular, Spiral, Multi-Index Slide Rule and Four Figures Logarithmic Decimals Scale Table is mounted in a wooden frame. It has single cycle number and common logarithm scales on an outer ring, and a 10 -spiral 421cm long-scale running outwards from near the center of the paper scale surface. Three brass indicators facilitate the calculations. Pickworth [47] and Cajori [9] also mention spiral slide rules by Schuermann in (c1896) and Fearnley (c1900), but give no details.

The first pocket watch style circular slide rule with a segmented long-scale - the Boucher or Calculigraphe - was designed by Henri Chatelain in France in the 1870s [27]. It has a $30-\mathrm{cm}$ long-scale made up on 3-concentric rings. The Calculigraphe was made or sold by several different engineering instrument specialists including: Keuffel \& Esser [36] and Dietzgen [16] in the US, W.F. Stanley, "Manlove, Alliott \& Fryer", and J.F. Steward in The UK, J-B Rehan and Pedos S.A. in Switzerland, and Henri Chatelain, and others in France [26]. Models are known to be marked H-C, F-C., A.F., E.D. Co., Stanley, Manlove, "Manlove, and Alliott \& Fryer", and K\&E Co.. The example in my collection is marked H-C, and came with a K\&E instruction pamphlet that is labeled: "Boucher Calculators (Calculigraphs.) - Keuffel \& Esser Co. - 127 Fulton Street, New York". Another pocket watch slide rule with a
segmented long-scale that appeared in the late 1800s is the Mechanical Engineer. This pocket calculator has two concentric ring segments, and came in several sizes including models with $17-$ and $24-\mathrm{cm}$ scale lengths. Examples are known with and without the Mechanical Engineer marking. One example is marked "SWISS" [15], lending a clue to the country of origin. The Mechanical Engineer was sold by W.F. Stanley, The Scientific Publishing Co, and others.

The first mention of a cylindrical slide rule that I found in Cajori's [9] history (other than Delamain's fantasy "Great Cylinder") is one attributed to Hoyau (1816) in France. However, no details are given. According to Cajori, both
J.D. Everett (a Scottish $19^{\text {th }}$ century maker of slide rules), MacFarlane (c1842) and Amedee Mannheim (the most influential of French slide rule designers in the $19^{\text {th }}$ century) also designed cylindrical slide rules. However, we do not have any details for these either.

Arguably, one of the most innovative longscale designs was the helix scale laid out on a cylinder. The most widely known of this type of slide rule was the Fuller spiral "Calculator" made by Stanley of London, and invented by George Fuller in 1878. The Fuller "Calculator" has a
 scale length of almost
$12.8-\mathrm{m}$ that winds around a $7.6-\mathrm{cm}$ diameter cylinder - 20 twenty times. Two brass index pointers facilitate the calculations. It came in several different models. It was well described by Feely and Schure [22]. Fuller may also have designed a "Midget" model having a scale length of about $5.08-\mathrm{m}$.

According to Cajori [9], other cylindrical slide rules with spiral scales were also designed by Mannheim, MacFarlane, Everett, G.H. Darwin and Prof. R.H. Smith from the middle to the late $18^{\text {th }}$ century. Little is known about the Mannheim, MacFarlane, Everett, and Darwin devices. However, several examples of the Smith Calculator are known with scale lengths of $102-$ and $107-\mathrm{cm}$. See Figure 5, for example.


Figure 6: Paisley tape calculator

Another innovative development in long-scale technology in the US was the cylindrical slide rule patented by an American civil engineer, Edwin Thacher [21], in 1881. Thatcher broke a double calculating scale into 40 segments, each $46-\mathrm{cm}$ long, and each repeating half of the preceding scale. The cylinder slides inside a sleeve of 20 parallel rods, each having two sections of the double scale (matching 2 sections on the cylinder). The effective length of the Thacher long-scale is $9.14-\mathrm{m}$. Thacher cylindrical slide rules were made in England until c1900. Thereafter, they were made by Keuffel \& Esser in the United States.

Two other makers of cylindrical slide rules got their start at the end of the $19^{\text {th }}$ century, the Swiss companies Daemen-Schmid (1896) - which later became Loga - and Billeter (1886). These companies were most active in the $20^{\text {th }}$ Century. Their cylindrical slide rules will be discussed in a later chapter.

One other long-scale slide rule innovation reported by Cajori [9] takes the form of a tape that is taken up on a spool or spools. The "idea is to place the logarithmic line upon a continuous metallic tapes, wound from one roller or spool upon another as in instruments by Darwin (1875) and Tower (1885). Cajori provides no details for these makers. A later version of this type of slide rule developed by J.R. Paisley in 1939 (Fig. 6). Another was patented by Silvio Masera in 1902 [32].

## Long-Scale Slide Rules in the 20thCentury

Important sources for details on "long-scale" slide rules in the more recent history were the wonderful books on slide rules by Peter Hopp [26] and Dieter von Jezierski [68]. Other sources included offerings from many other slide rule collectors and my own collection of slide rules. These are the long-scale slide rules that the collector has the greatest chance of finding. In this section, I have broken the discussion down for the various formats of slide rules. Tables I and II summarize the results.

## Slide Rules with Linear Formats

This category focus primarily on segmented scale linear slide rules, and does not cover slide rules with singlesegment $50-\mathrm{cm}$ scales as there are just too many to consider here. However, there are two single-segment straight slide rules with scales longer than $50-\mathrm{cm}$ that deserve mentioning. One of the first (c1904) in the $20^{\text {th }}$ century that comes to mind is the Scofield-Thacher Engineer's slide rule [55]. It has several scales designed for the structural engineer, including a $56-\mathrm{cm}$ long single cycle calculating scale. Another is the Nestler Reitz, made in the 1930s, that had a single-segment scale with a length of 1-m [26 \& 68].

One more long-scale slide rule that deserves recognition is the "Texas Magnum" slide rule constructed by Skip Solberg and Jay Francis [62] in an aircraft hanger on February 28, 2001. It is 107 -m long in one segment, having a precision of nearly 6 -digits. While this slide rule was not intended for commercial production, it is recognized by the Guinness Book of Records as the longest slide rule ever made.

In the $20^{\text {th }}$ century, slide rule makers introduced several new linear slide rules
 with segmented calculating scales. For instance, the

Figure 7: Post Ritow long slide rule
Nestler [68]
Precision \#27/9 (also listed as 27/a) model was a $50-\mathrm{cm}$ slide rule which had a pair of calculating scales broken into two segments, each $50-\mathrm{cm}$ in length. The first segment runs from 100 to the square root of 1000 , and the second segment runs from the square root of 1000 to 1000 . There are a pair of $1^{\text {st }}$ segments at the upper margin of the slide and a pair of $2^{\text {nd }}$ segments at the lower margin of the slide. The effect of this arrangement was to give a calculating scale of $100-\mathrm{cm}$ length. The Nestler Precision slide rule also came in $15-\mathrm{cm}$ and $25-\mathrm{cm}$ scale lengths having calculating scale lengths of $30-$ and $50-\mathrm{cm}$ respectively. Table 1 shows that other slide rules with 2 -segment scales were made (or sold) by Wichmann, Unique, Faber-Castell, Dietzgen, Roos, and Favor.

In my collection (Fig. 7), I have a $50-\mathrm{cm}$ (German made) Post slide rule with a pair of $100-\mathrm{cm}$ scales, each made up of 2 segments - half of each pair on the stators and half on the slide. This slide rule is interesting because the half scale sections on the slide are inverted - something like the CI scale on a modern slide rule. In 1910, Pickworth [47] described a similar slide rule and named it the "Long" slide rule. The example in my collection has the Dennert \& Pape patented screw adjustment system for regulating slide friction. A German patent was issued for this system in 1903, so the 'Long" scale slide rule mentioned by Pickworth and the example in my collection could be the same, with the German maker, perhaps, being Denert \& Pape. Post may have sold this as a Ritow model \#1466 slide rule [50], which was listed in Frederick Post catalogs in the 1920s.

The $25-\mathrm{cm}$ Unique Pioneer Long-scale [6] and the $50-\mathrm{cm}$ Hemmi \#201 [19] slide rules take the segmented scale innovation to another level. Both of these slide rules break the calculating scales into 4 pieces each, the resulting
scale length being $100-\mathrm{cm}$ for the Unique Pioneer and $200-\mathrm{cm}$ for the Hemmi \#201. The C scale is broken into four equal length segments on the slide and four matching D-scale segments on the lower stator. For this case, the number of digits that can be resolved is further improved to about four. However, it begins to get a bit tricky in deciding on which scale to read the result. One either calculates the approximate result in their head, or resorts to making a calculation with a normal pair of C and D scales before using the segmented C and D scales.

The Hemmi \#200 [68] is a $41-\mathrm{cm}$ duplex slide rule that breaks the scale into even more segments. It breaks the C and D scales into six sections each, giving an effective $244-\mathrm{cm}$ scale length. As for the Hemmi \#201, one must be adept at calculating the approximate result in ones head, or resort to a normal set of calculating scales to get the approximate result so that one knows which scale to read the result on.

## Gridiron Slide Rules

As we have learned, the earliest of the gridiron slide rules found was the 10 -segment, $305-\mathrm{cm}$ total length, slide rule attributed to William Nicholson (c1797) [9].

The Hannyngton "Extended" gridiron slide rule, which was first made in the late 1800 's, continued to be sold into the 1920s. Cherry's gridiron "Calculator" was also sold into the early 1900s. Proell's gridiron "Pocket Calculator" appeared at the turn of the $20^{\text {th }}$ century. It was similar to the Cherry Calculator, excepting that the scales on the sliding part (clear celluloid) run from right to left, much like the inversed scale on a common slide rule. Multiplication is accomplished by placing a needle to fix the position of intermediate results in multiple calculation problems. Index marks on the fixed lower card enable square and cube roots to be extracted.

An instruction manual issued in 1909 by the Kolesch Co. [46] listed a gridiron slide rule called the "Calculigraph" or "Australian Slide Rule". It was printed on cardboard. The scales are printed on $13-\mathrm{cm}$. x $28-\mathrm{cm}$. "cardboard" in two ranks of 22 parallel segments to form a 2 -cycle calculating scale. A sliding transparent "bridge" with a one cycle segmented scale is used to make the calculations. The scale length was about $5.5-\mathrm{m}$. According to the advertising material, this slide rule had a max error $=1$ in 5000 , or nearly five digits precision.

The Gilson Slide Rule Company is better known for its circular slide rules, but it appears that very early in this company's existence (ca. 1915) it sold a linear pocket gridiron slide rule that broke the calculating scales into 14


Figure 8: Logaritmal gridiron slide rule sections. An advertisement in the instruction manual [49] for the Richardson Direct reading slide rule shows the Gilson "Pocket Slide Rule" having 14 -section calculating scales for a total scale length of about $178-\mathrm{cm}$. The scales are printed on "heavy water-proof Bristol, " a cardboard like material. The price was 50 cents. The only mention of this slide rule that I have been able to find is in the Richardson and Clark instruction manual. It is uncertain if any copies have survived.

The Cooper (20-segments, 250cm length) is an interesting variant of a gridiron slide rule sold in the early 1900s. This slide rule was described in detail in an article by Bennett [7]. A single 20-section scale was laid out on a white celluloid sheet laminated to a mahogany board, with the scale on 20 parallel lines, each about $13-\mathrm{cm}$ long. The four corners of the block of scales are marked with special indicator marks labeled with the number 100. A separate
clear celluloid sheet with indicator matching slides over the calculating scale. The calculations are made using the appropriate corner indicator mark and a weighted pointer that freely slides over the clear celluloid sheet.

Hopp [26] mentioned that Gladstone's "Cross Gauge" was much like the Everett gridiron, but gives no details other than it had a scale length of greater than $10-\mathrm{m}$. It was sold in the 1920s. Two other gridiron slide rules available in the $20^{\text {th }}$ century have come to my attention. One is the Marotti "Lagartabla" gridiron. This appears to be of eastern European origin. It has 27 -segment base (repeating) scale printed on a metal sheet, and a 14 -section sliding metal piece with slots to reveal the scale beneath it. The scale length is about $240-\mathrm{cm}$. One was sold at auction at IM2000 [28]. Unfortunately for me, I was not alert enough during this auction to prevail. The other gridiron slide rule, the "Logaritmal" (Fig. 8) was designed by Văclav Jelinek. Reinhard Atzbach [3] reported details for the "Logaritmal" on his Internet site, including download images for construction of this gridiron slide rule. It breaks the scale into ten segments, having a total length of $1.5-\mathrm{m}$.

## Slide Rules with Saw Tooth Formats

One more interesting idea that surfaced in the $20^{\text {th }}$ century to improve slide rule precision was the incorporation a 'vernier' into the calculating scale by a kind of saw tooth arrangement. Babcock [4] reported on two different approaches. One by A.N. Lurie developed in 1910 used diagonal lines drawn from the bottom of one scale division to the top of the next. This diagonal line, in combination with a series of horizontal crossbars on the cursor, allows the user to divide the space between the divisions into 10 parts. Lurie applied his method to an ordinary $25-\mathrm{cm}$ Mannheim slide rule, but Richardson employed a similar concept to a gridiron type scale. Richardson (ca. 1918) used a method designed by Yu Wang, whereby a kind of tent or triangle is laid out between gradations. In Yu's design, five parallel horizontal lines are drawn within the triangle. The reading of the cursor hairline is then determined by where it crosses the point of intersection of one of the horizontal lines and one of the diagonal lines. This same concept was employed on the Appoullot "Logz" circular slide rule [54], but in this case the 'tent' between gradations is formed by 10 short lines drawn normal to the scale direction. In this case, the 'tent' is located in the space outside of the calculating ring where there is more room. The Appoullot slide rule is also interesting because it incorporates a spiral scale.

## Charts and Table Slide Rules

The Goodchild and the LaCroix and Ragot calculating charts are interesting variations of the gridiron type slide rule, but without a sliding piece. The Goodchild Mathematical Chart and its accessory triangular (tallying) rule were sold by K\&E [36] for a short time in the early 1900's. The chart broke the number scale down into 100 parallel segments on a single folded card stock sheet. The total scale length is over $16-\mathrm{m}$. Each line is numbered at the beginning and end with the first two digits of the mantissa. The balance of the logarithm is represented by the distance along the segment. Every fifth gradation along the segment is labeled with the number represented in the logarithmic
 scale by the particular segment and distance. The triangular tallying scale acts

Figure 9: Goodchild tallying rule like a bridge to enable the calculations. One side has scales and a slide to add (or subtract) the first two digits of the mantissa of the numbers in the operation. This gives the initial two digits of the line on which the result will be found. The other two sides each have a series of equally spaced gradations and very short slides that are used as index mrkers to keep track of the distance of the reading from the left edge of the column. Figure 9 shows the triangular rule on a reproduction of the

Goodchild chart. A few of the Triangular Rules are known, but I have been unable to find an original copy of the Goodchild chart.

The LaCroix and Ragot Graphic Table [37] is a very long (111-m) 5-place graphical table form of a gridiron slide rule. The table is laid out in 1000 lines over 40 pages in a book format. This graphical table is used much like a table of logarithms, the operations being done by adding (or subtracting) logs of numbers in the normal way. It cannot be used with a pair of dividers or tallying rule because the scale segments are not divided into equal lengths. The advantage of this table is that it is much more compact that a conventional 5-place table of logarithms.


Figure 10: Dumesnil, Regle Universelle Deposee

Another printed graphic table that I acquired recently is one printed on a rule by C . Dumesnil, the "Régle Universelle Déposée". It has a 2-m long-scale laid out on eight parallel segments on paper on the front and back of a boxwood rule. It appears that one uses dividers to make the calculations. I have seen two examples of this rule; one was in an ornate brass souvenir case for a Paris Exposition. The example in my collection (Fig.10) is signed "Médaille A L'Exposition Universelle de 1900"

One other chart type slide rule deserves attention. "The MacMillan Table Slide Rule" [5] is unique among slide rules in that it takes the form of a table of discrete logarithms, not an analog scale of logarithms. There are four tables laid out on card stock, one each for number, logarithm, sine and tangent operations. Each table has 2-cycles of data in a 201 -line by 20 -column format. The calculations are performed using cardstock slides which have a matching format, but with only half the width.

One more possible graphic chart type logarithmic calculator produced in the $20^{\text {th }}$ century, the Knowles "Calculating Scale.", was reported by Cajori [9]. It is uncertain if this was of the graphic chart type or a gridiron calculator.

## Circular Slide Rules with Circular Scales

Following the innovations of the earliest slide rule makers, many different makers of circular slide rules emerged in the 1900's. Table II shows a few examples of circular slide rules with long-scales, including singlering circular slide rules with long-scale lengths from about $50-\mathrm{cm}$ (German Norma Graphia 190) up to about 1-m (East German Tröger). The popular Gilson Binary and Atlas (made in the US) circular slide rules had longscale lengths of about $64-\mathrm{cm}$ on their perimeters. None of the circular slide rules of the $20^{\text {th }}$ century had long (single-ring) scales approaching the lengths (1.4- to 1.5 m ) of the $18^{\text {th }}$ century Scott and Clairault circular slide rules.


Figure 11: Demster RotaRule

## Circular Slide Rules with Concentric Ring Scales

The Boucher (Calculigraphe) continued to be available into the first quarter of the 20th century. It had a 3-ring scale length of about $30-\mathrm{cm}$. In the early $1900 \mathrm{~s}, \mathrm{~K} \& E$ introduced another pocket watch style circular slide rule with a 3ring scale - the Sperry. It had a scale length of about $32-\mathrm{cm}$. In the 2 nd and 3 rd quarters of the 20 th century, Fowler [27] produced three different long-scale slide rules with a pocket watch format. They were the 'Long scale" (6-ring, $76-\mathrm{cm}$ ), the "Long Scale Magnum" (6-ring, 127-cm), and the "Jubilee Magnum Extra Long Scale" (11-ring, 185-
$\mathrm{cm})$. These slide rules were very popular in England and many examples are in collections. The Dempster RotaRule [60] was one of the most complex of the concentric ring circular slide rules. It had many of the scales common to the $\log \log$ duplex slide rule, in addition to a $4-\mathrm{ring}, 127-\mathrm{cm}$ long-scale (see Figure 11). Pickett and Boykin made copies of the RotaRule. The British company, Unique, also showed in their catalogs, a $5-\mathrm{ring}, 127-\mathrm{cm}$ long-scale "Dial Calculator" [26]. The longest scale concentric ring circular slide rule that I found reference to was the Sexton "Omnimetre \#6 (Companion)" [2] circular slide rule. It had 20-concentric rings and a scale length of about 411-cm. No examples are known to me.

Circular Slide Rules with Spiral Scales


Several new circular spiral slide rules appeared in the $20^{\text {th }}$ century. The Gilson Atlas was probably the most widely known make of this type of slide rule in the United States. It came in three different versions [1]. The standard Gilson Atlas slide rule had a scale length of about $10.7-\mathrm{m}$. The spiral winds 25 revolutions on the $21-\mathrm{cm}$ diameter disk. The Atlas has an extra ring at the outer edge of the disk that contains one complete calculating scale. One first makes the calculation on this outer ring to obtain the result to 3 to 4 digits, and then repeats the calculation on the spiral scale to get the result to about five digits. The precision of the readings is nearly the same at both ends of the scale. This is the result of the increasing diameter of each winding. This advantage is more pronounced on spiral scales with large diameters and large numbers of windings. Gilson also made two early versions of the Atlas, one sometimes referred to as the 'square' Atlas and the

Figure 12: Gilson Atlas
other a smaller diameter version (Fig. 12) of the 'square' Atlas, that had 30 windings, and scale lengths of nearly $14-\mathrm{m}$ and $11.9-\mathrm{m}$ respectively. The Gilson "Atlas" slide rules could resolve calculations to about five digits. The 'square' Atlas has a scale length even longer than the scale lengths of the Fuller and Thacher cylindrical slide rules.

The lesser-known "Ross Precision Computer" (Fig. 13) has a spiral calculating scale, much like that of the common Atlas slide rule. It has 25 windings, and the scale length is $9.14-\mathrm{m}$. It can be read to about the same precision as the Atlas. The Ross, however, has a unique feature. It has a rectilinear slide rule attached to a radial arm on the pivot point. One first


Figure 13: Ross Precision Computer makes the calculation on the straight slide rule, and then on the spiral scale using two celluloid indicators. The position of the cursor on the straight slide rule lines up with the appropriate winding on the spiral scale to obtain the result. Examples of the Ross slide rule are quite scarce - with maybe 10 to 20 copies in collections. Many of these are in poor condition because of the unstable metals used in their manufacture. I gathered details from an example in my collection that is made of stainless steel and brass.


Others including the Appoullot "Logz" [54], Logomat " $V$ \& $G$ Nr.816" [53], Alro "Commercial" [52] and Concise "M.V. Douglas" [42] spiral slide rules have shorter scale lengths ( $68-, 110-, 150-$ and $152-\mathrm{cm}$ ) and have fewer windings ( $2,3,6$ and 10) than the Atlas and Ross slide rules. The precision of the readings made with these rules is about one digit less than for the Atlas and Ross models.

The Appoullot Logz (Fig.14) deserves a special mention for its artistic qualities. Its spiral scale winds in a free hand pattern, and actually jumps over another scale. It also has a vernier scale for obtaining more precise readings and decorative transparent celluloid indicators. I also cannot go without showing a picture of the Dutch Alro Commercial spiral slide rule. It can be seen in the familiar hinged Alro case in Figure 15.

## Cylindrical Slide Rules with Helical Scales

The Fuller helix "Calculator" gained its popularity in the $20^{\text {th }}$ century, and is one of the most widely known of the cylindrical slide rules made. Stanley of London started making them in the 1880 's, and continued making them right up until the electronic pocket calculator brought an end to slide rule use in the early1970's. More than 14,000 were made [22]. The Fuller has a scale that winds around the cylinder 20 times to give a total scale length of about $12.7-\mathrm{m}$. One can resolve the readings to a precision of 5 digits at the left index and 4.5 digits at the right index, not quite as good as for the Gilson 'square' Atlas, but a little better than the common Gilson Atlas slide rule. Dobie, an Australian company, may have produced Fullers under license to Stanley, the maker of Fuller "Calculators" in the 1970s. Frederick Post Co., an American supplier of engineer's products, listed a cylindrical slide rules with a $12.7-\mathrm{m}$ long helix scale (the same length as the Fuller) in their catalogs in the 1920s under the Ritow line of slide rules [48]. It is described as being about


Figure 15: Alro Commercial $7.6-\mathrm{cm}$ diameter and $30-\mathrm{cm}$ long, with a $12.3-\mathrm{m}$ helical scale, and a magnifying glass to improve accuracy. However, no examples of the Ritow cylindrical slide rule are known.

Other production cylindrical slide rules with helix scales include the well-known Otis King pocket cylindrical slide rule, and the less common R.H. Smith Calculator. The Otis King model ' $K$ ' has one double scale with 40 -windings and a second single log cycle scale with 20 -windings about a nominal $2.5-\mathrm{cm}$ diameter cylinder [25]. A sliding cursor sleeve facilitates the calculations. The Otis King scale length is about $1.7-\mathrm{m}$, and it can be read with a precision of about four digits. Examples of the Otis King are very common. They were made from the 1920's right up until the early 1970 's. Because they are so common, they are, perhaps, the first of the cylindrical slide rules to make it into a collection.


Figure 16: Lafay: Helice a Calcul

The R.H. Smith Calculator was described by Weinstock [70]. A description of this cylindrical slide rule also appears is in the $11^{\text {th }}$ edition (1910) of Pickworth's [47] book. Weinstock's R.H. Smith slide rule has a cylinder diameter of about $1.3-\mathrm{cm}$ and a scale length of about $1-\mathrm{m}$, which is contrast to the $1.9-\mathrm{cm}$ diameter cylinder and $1.3-\mathrm{m}$ scale length reported in the Pickworth book. The cursors are also different for these two models, the Weinstock version having 2 brass rods, much like the Fuller cylindrical slide rule, whereas the Pickworth version (see Fig. 5, for example) has an actual sliding cursor something like that of the Otis King, only much shorter.

I have one other cylindrical slide rule with a spiral calculating scale in my collection that deserves mention. It is marked "Helice a Calcul - No.2", and was made by A. Lafay of Neuville S/Saone in France. The spiral scale winds 50 times around a $4-\mathrm{cm}$ diameter tube to give a total scale length of about $2.5-\mathrm{m}$ and a precision of four digits. A sliding celluloid sleeve and three celluloid cursors facilitate the calculations. Figure 16 shows the Lafay slide rule. I have been unable to find any other information on this slide rule and its maker.

Andrew Davie [13] showed a picture of a "model calculating cylinder" cylindrical slide rule designed by the Russian, Alexander Schukarev ca. 1910 that has a design like the Lafay cylindrical slide rule. It appears to have about 80 windings. No other details for this slide rule are available.

## Cylindrical Slide Rules with Linear Segment Scales

The most widely known of the cylindrical slide rules with linear segment scales in the US is the Thacher Calculator patented by Edwin Thacher in 1881 and sold by K\&E starting in the late 1800s. As described earlier, the Thacher is essentially a $9.14-\mathrm{m}$ long-scale gridiron slide rule in a cylindrical format. The Thacher slide rule was made from the 1880's right up into the 1940's. Many copies of this slide rule are known to collectors as nearly 8,000 were made.

The Loga Rechenwalze (and its Swiss and German cousins) was the most widely known cylindrical slide rules in continental Europe. The Loga cylindrical slide rule is similar to the Thacher in its operation, excepting that the cylinder is fixed and the segmented scales on a sleeve slide on the cylinder. The sliding sleeve has a length about half of that of the cylinder, and the segments on it make up a single scale. Each segment on the cylinder repeats half of the previous segment to facilitate the calculations. This is a Swiss made calculator that has its origins in the late 1800s under the brand name Daemen-Schmid, and becoming Loga in 1915 [32]. It came in many different diameters and long-scale lengths - including lengths of 1.2-, 2-, 2.4-, 7.5-, 10-, 15- and 24-meters [33]. The 24-m Loga has the longest scale length of any slide rule that I reviewed (other than the printed tables).

The company Billeter started making cylindrical slide rules earlier than Daemen-Schmid in 1888 [32]. Members of the Billeter family became involved in the National Cylindrical Slide Rule Ltd. in about 1917, when the company Billiter was dissolved. Billeter is known to make cylindrical slide rules (similar to the DaemenSchmid / Loga rechenwalzen) with 2- and 10meter long-scale lengths. National made cylindrical slide rules with long-scale lengths of


Figure 17: Tröger 7.37 Rechenwalze 8 -, 10- and 16-meters [33].

Similar cylindrical slide rules were also marketed by Nestler, a German maker of slide rules. Joss [33] lists a 1.6-m Nestler rechenwalze, and von Jezierski [68] lists two additional models, with 3.75- and $12.5-\mathrm{m}$ long-scale lengths. I have examples of the latter two models in my collection.

The East German company - Tröger - made two different cylindrical slide rule models $\{66]$ with scale lengths of 5and 25 - Bavarian feet ( $1.46-$ and $7.37-\mathrm{m}$ ). Tröger's DRGMs for cylindrical slide rules were granted in the early

1920s, but it is uncertain just when they started making cylindrical slide rules. The Tröger rechenwalzen (see Fig. 17, for example) have a unique scale arrangement. The segments on the cylinder repeat 2 times and those on the sliding sleeve repeat once, in contrast to the arrangement on the Loga, National and Nestler models where the scales on the cylinder repeat just once and those on the sleeve do not repeat.

One other cylindrical slide rule of this type was brought to my attention by Roger Shepherd [59]. That is the Japanese Kooler Calculator by Muto Giken. It is a modern version (perhaps 1960's) with 50 double scale segments laid out longitudinally on a 1.85 -inch diameter by 11.5 inch long tube. The sliding sleeve has three ring-indicators (like the Lafay "Helice a


Figure 18: Cylindrical slide rule (unknown maker) Calcul') with hairlines rather than a set of calculating scales. The effective long-scale length is about $4.9-\mathrm{m}$.

Joss [33] also mentions: the 2-meter "Reciloga" made by Edmund Schneider of Munich, Germany; the 14.4-meter "Numa" rechenwalze of unknown Swiss origins; and some "no name" cylindrical slide rules. I have one "no-name" cylindrical slide rule in my collection (Fig. 18) that has an identical scale arrangement to the $1.46-\mathrm{m}$ Tröger. It is marked with the German registration number DRGM 639848.

Hopp [26] mentions one other cylindrical slide rule sold by Reiss, but gives no details. It may have been from one of the above makers, and relabeled under the Reiss name.

## Tape Slide Rules

The only long-scale slide rule that I found with a tape format was one copyrighted by Paisley in 1939. This slide rule has two continuous scales placed on side-by-side ribbons that wrap around spools at each end of the device. The scales are read through a window in the case. The Paisley slide rule was briefly described by Feely [20]. The scales on the ribbons are positioned relative to each other by turning knurled knobs at one end of he device. The Paisley "Calculator" (Fig. 6) has a scale length of about $51-\mathrm{cm}$. Also, according to Hopp [26], The German slide rule maker / reseller Wichmann listed a measuring tape slide rule, with a scale length of $50-\mathrm{cm}$, in their catalog in 1938. No details are available, but it could be relabeled Paisley "Calculator".

One other long-scale tape slide rule was mentioned by Joss [32]. It was attributed to Silvio Masera, Winterthu, "Rechenstab mit Endlosband", from a 1902 German patent, but it is not known if examples were produced.

## Some Observations and Conclusions

Long-scale slide rules evolved from the beginnings of slide rule technology in the 1600 s , well into the $20^{\text {th }}$ century. The longest scale production slide rules were the spiral and cylindrical types. The 24-meter Loga cylindrical slide rule is the longest production slide rule made. In my earlier report on long-scale slide rules [11], I showed that it could provide results to about five digits. The longest spiral scale slide rule made was the Gilson "square" Atlas. It had a scale length of about 14-meters, and could also give results to about five digits.

The spiral scale slide rules had several distinct advantages over the cylindrical slide rules. They were more compact and could fit in a desk drawer rather than on a desktop. They were also obviously less expensive to produce. The spiral slide rule was also more precise at the right end of the scale because of the expansion of the scale as the diameter increases. For example, the precision on the "square" Atlas spiral slide rule is about the same at both ends, whereas for the Loga cylindrical slide rule, it is about one half digit less at the right end. Furthermore, the spiral slide rules are easier to use. One wonders why so much effort was put into developing and producing cylindrical slide rules given the great advantages of the spiral slide rule and its early invention.

I would have liked to include more pictures of long-scale slide rules in this paper. However, for saving space considerations, I have not included pictures of the Thacher, Fuller, Otis King, Fowlers, etc. because they are widely known from the literature. I have included pictures of the more scare and rare long-scale slide rules.

Finally, I would, also, have liked to include images of the Sutton, Brown and Dixon spiral slide rules in this paper, but time constraints did not allow me to obtain the appropriate license. When in London, you can see these very important slide rules on display. The Sutton and Brown slide rules date to the very early period of slide rule making - ca.1660. The amount of detail in working the spiral scales on these slide rules is remarkable. One can almost feel the touch of William Oughtred, the inventor of the long-scale spiral slide rule, when examining these slide rules.

## Acknowledgements

I would like to express my appreciation to those who have provided me information on long-scale slide rules. These include; Collin Barnes, Jim Bready, Bob DeCesaris, Bobby Feazle, Mike Gabbert, Hermann van Herwijnen, Heinz Joss, John Kay, Klaus Kuehn, Guter Kügel, Wayne Lehnert, Dick Lyon, Andreas De Man, John Mosand, Bob Otnes, Dick Rose, Paul Ross, IJzebrand Schuitema, Roger Shepherd, Francis Wells, and Thomas vander Zijden. I would also like to acknowledge the slide rule egroup for rising to the occasion several times to answer my questions. I would also like to acknowledge the permission given by the Science Museum in London to publish pictures of the Sutton, Brown and Dixon spiral slide rules. This paper is an update of a report previously published in the Journal of the Oughtred Society (v.8, n.1). I am grateful to Bob Otnes, publisher of the JOS, for giving permission to publish this paper in the proceedings of IM2003.

## References

1. Aldinger, Henry and Edwin Chamberlain, "Gilson Slide Rules - Part II - The Large Rules", The Journal of the Oughtred Society, v.9, n.2, Fall, 2000, p. 47-58
2. Alteneder, Theodore \& Sons, "Price List of Sexton's Omimetre", Carlin Brothers

Press, Philadelphia, 1897.
3. Atzbach, Reinhard, Internet web site http://rover.vistecprivat.de/~ratz/rechsamm/ logaritmal.htm, provided details for "Logaritmal" gridiron slide rule, designed by Ing. Dr. Văclav Jelinek, M.-Ostrau - M. Ostrava.
4. Babcock, B.E., "Two Noble Attempts to Improve the Slide Rule", The Journal of the Oughtred Society, v.4, n.1, March, 1995, p. 41-47.
5. Ballantine, J.P., The Macmillan Table Slide Rule, The MacMillan Co., New York, 1931.
6. Barnes, Colin, personal communication; provided details of Unique Pioneer Long-scale slide rule.
7. Bennett, A., "The 'Cooper’ 100-Inch Slide Rule", The Journal of the Oughtred Society, v.3, n.1, March, 1994, p. 18-19.
8. Bready, James, personal communication; sent picture files for the Wichmann 50 / 100 and Tavernier-Gravett longscale slide rules.
9. Cajori, Florian, A History of the Logarithmic Slide Rule and Allied Instruments, The Astragel Press, Mendham, NJ, p. 1-136, originally published in 1910.
10. Cajori, Florian, A History of Gunter's Scale and the Slide Rule, The Astragel Press, Mendham, NJ, p. 137-164, originally published in 1910 .
11. Chamberlain, Edwin, "Long-Scale Slide Rules", The Journal of the Oughtred Society, v.8, n.1, Spring, 1999, p. 24-34.
12. Chamberlain, Edwin, "Three American Printed Page Logarithmic Calculating Scales, The Journal of the Oughtred Society, v.11, n.1, Spring 2002, p. 8-13.
13. Davie, Andrew, Internet site: http://www.taswegian.com/TwoHeaded/
cylinderUSSR.jpg, shows coarse photograph of cylindrical slide rule "patented or registered" by Prof. Alexander Schukarev, "Модель счетного цнлиндра" translates as model calculating cylinder.
14. DeCesaris, Robert, personal communication, provided details for small and large Hannyngton gridiron slide rules.
15. DeCesaris, Robert, "The Mechanical Engineer", The Journal of the Oughtred Society, v.7, n.1, Spring, 1998, p. 23-24.
16. Dietzgen Co., Eugene, Catalogue of Drawing Materials and Surveying Instruments, $17^{\text {th }}$ edition, 1904-1905, p.174. sold Calculigraphe as "Boucher Calculator" marked E.D. Co., p. 174
17. Encyclopedia Britannica, $3^{\text {rd }}$ edition, 1798, Plate CCLXXIII
18. Feazel, Bobby, "Palmer's Computing Scale", The Journal of the Oughtred Society, v.3, n.1, March, 1994, p. 917.
19. Feazel, Bobby, personal communication; provided photocopy of Hemmi 201 slide rule..
20. Feely, W., "The Paisley Slide Rule", The Journal of the Early American Industry Society, v.50, n.1, March, 1997, p. 113.
21. Feely, W. and C. Schure, "Thacher Slide Rule Production" The Journal of the Oughtred Society, v.3, n.2, Sept., 1994, p. P.38-42.
22. Feely, W. and C. Schure, "The Fuller Calculating Instrument" The Journal of the Oughtred Society, v.4, n.1, March, 1995, p. 33-40.
23. Girbardt, Werner and Werner T. Schmidt, Tröger Logarithmic Computing Cylinder, Internet site: www.unigreifwald.de, Institute for Mathematics and Technical Information, University-Greifwald,
24. Holland, Peter, "Printed Logarithmic Calculating Scale by Loewe", The Journal of the Oughtred Society, v.11, n. 1, Spring 2002, p. 14-16.
25. Hopp, Peter M, "Otis King Update", The Journal of the Oughtred Society, v.4, n.2, October, 1995, p. 33-40.
26. Hopp, Peter M. Slide Rules - Their History, Models, and Makers, Astragal Press, 1999, 310p.
27. Hopp, Peter M. Pocket-Watch Slide Rules of the World, Proceedings, $4^{\text {th }}$ International Meeting of Slide Rule Collectors, Huttwil, Switzerland, Oct.14-16, 1998, p.63-69.
28. International Meeting of Slide rule Collectors, Ede - The Netherlands, 22-23 September, 2000.
29. The Journal of the Oughtred Society, v.5, n.1, March, 1996, cover photograph, "The Oughtred circular slide rule by Ellias Allen".
30. The Journal of the Oughtred Society, v.5, n.2, October, 1996, Illustrations in the Centerfold, General Hannyngton slide rule, p. 54 .
31. The Journal of the Oughtred Society, v.8, n.1, Spring, 1999, cover photograph, "Spiral Slide Rule made by Bernardus Facini in 1710", courtesy of the Adler Planetarium, History of Astronomy Department, Chicago
32. Joss, Heinz, "Swiss Persons, Companies and Brands from the History and Present of the Slide Rule", $4^{\text {th }}$ International Meeting of Slide Rule Collectors - Huttwil, Switzerland, 14-16 October, 1998, p.53-58.
33. Joss, Heinz, "Drum Slide Rules - Slide Rules with Long Scales", Slide Rule Papers presented at the "Symposium on the development of Calculator technology", Greifswald, Germany, Sept. 2000, reprinted by the UK Slide Rule Circle, UKSRC, 2001, p. 1-22.
34. Joss, Heinz, "A Tablet Slide Rule by Julius Billeter, Zurich", Journal of the Oughtred Society, v.9.,n.2, fall 2000, p. 27
35. Keuffel \& Esser Co., Catalogue of Keuffel \& Esser Co., 26th edition, 1895, p.185.
36. Keuffel \& Esser Co., Catalogue of Keuffel \& Esser Co., $31^{\text {st }}$ edition, 1903, listed Calculigraphe (p. ) and Goodchild Mathematical Chart (p. )
37. La Croix, A. and C.L. Ragot , A Graphic Table Combining Logarithms and Anti-Logarithms, The MacMillan Co., 1925.
38. Loewe, N.N., Rechenscallen für numerisches und graphisches Rechen, Heft1: Logarithmosche Rechenscalen, Liebenwerda, Verlag des Technischen Versandgeschäfts R. Reiss, 1893.
39. Manley, Ron; Slide Rule Internet Site: http://www.sliderules.clara.net/collection/ nonstandard/0010-tg.htm, .shows Tavernier-Gravet long-scale slide rule.
40. Miller, R.C., "Nystrom's Calculator", The Journal of the Oughtred Society, v.4, n.2, Oct., 1995, p. 7-13.
41. National Institute of Standards and Technology Virtual Museum, "Internet Site:
http://museum.nist.gov/object.asp?ObjID=5", Gaithersburg, Maryland, 2003, shows Hassler Geodedic slide rule.
42. O'Leary, Michael, personal communication, provided scanned picture of Concise M.V. Douglas spiral circular slide rule.
43. Otnes, Robert, "Elmer A. Sperry and His Calculator", The Journal of the Oughtred Society, v.6, n.2, Fall, 1997, p. 19-22.
44. Otnes, Robert, "Notes on Frederick Post Slide Rules", The Journal of the Oughtred Society, v.7, n.1, Spring, 1998, p. 7-10.
45. Otnes, Robert, personal communication, Robertson sliding Gunter rule with $72-\mathrm{cm}$ scale.
46. Petri-Palmedo, D., How to Use Slide Rules, Kolesch \& Co., New York, 1909.
47. Pickworth, C.N., The Slide Rule - A Practical Manual, $11^{\text {th }}$ edition, Colstons Limited Printers - Edinburgh, 1910.
48. Post Co., Frederick, Catalog, 1925, p. 146.
49. Richardson, Geo. W. and J.J. Clark, The Slide Rule Simplified, $6^{\text {th }}$ edition, The Technical Supply Co., Scranton, Penn., copyright 1918
50. Ross, Paul and Ted Hume, "Slide Rules of Frederick Post", The Journal of the Oughtred Society, v.9, n.2, Fall, 2000, p.37-46.
51. Schmidt, Werner H. http://www.uni-greifswald.de/~wwwmathe/RTS/rs010.html. Tröger Logarithmische Rechenwalze
52. Schuitema, Ijzebrand, "The ALRO Circular Slide Rule", The Journal of the Oughtred Society, v.2, n.2, October, 1993, p.24-37.
53. Schuitema, IJ, "Logomat Werbeartikel Logomat Advertising Articles", The Journal of the Oughtred Society, v.4, n.1, March, 1995, p. 29-32.
54. Schuitema, Ijzebrand, "The Appoullot Circular Slide Rule", The Journal of the Oughtred Society, v.4, n.1, March, 1995, p.48-52.
55. Schure, Conrad, "The Scofield-Thacher Slide Rule", The Journal of the Oughtred Society, v.3, n.1, March, 1994, p.20-25.
56. Science Museum, London, Society \& Picture Library, Circular slide rule, 1600-1699, Photograph of John Brown spiral slide rule, "Circular slide rule with two brass arms and an astronomical quadrant engraved on the back", SCM / MAT / B730376B.
57. Science Museum, London, Society \& Picture Library, Mathematical Instruments, 1600-1699, Photograph of Henr Sutton spiral slide rule, "Spiral slide-rule, brass, engraved: 'Henr Sutton fecit, 1663' ", SCM / MAT / B731079B.
58. Science Museum, London, Society \& Picture Library, Mathematical Instruments, 1800-1900, Photograph of T. Dixon spiral slide rule, "Combined circular and spiral slide rule, in a wooden frame, 1882. combined circular and spiral slide rule made by T Dixon with a treatise on its use. ", Picture number: MATC100603.
59. Shepherd, Rodger, provided photos of Muto Giken Kooler Calculator from his collection.
60. Shepherd, Rogerd, "The Dempster RotaRule", The Journal of the Oughtred Society, v.7, n.1, Spring, 1998, p.4-6.
61. Skinner, Inc., Science \& Technology Auction, Sale 1790, Bolton, MA, July 19, 1997, auctioned a Nystrom's Calculator.
62. Solberg, Skip, Internet site: http://www.slideruleguy.com/fwst.htm, details the construction of the "Guinness Book of Records" longest slide rule in the world, length $=107-\mathrm{m}$.
63. Stanley. William_S., personal communication; Stanley has Boykin RotaRule in his collection; the maker Bernard C. Boykin is his uncle.
64. The Gemmary, posted at auction one R.H. Smith Calculator (Pickworth type) on Feb.21, 2002.
65. Tichy, Anton, Graphische Logarithmen-Tafeln (Graphic Logarithmic Table), Vienna, Verlag des Österreichen Ingenieur und Architekten-Vereines, 1897.
66. Tröger, K. Emil, Rechenscheiben-Rechenwalzen - Prozentrechen Leich gemacht-durch die Rechenscheibe Beschreibung und Gebrausnsanweisung, Mylau-Vogtl, 4.p
67. van Herwijnen, Herman, Slide Rule Catalog, on CDROM, v.5.1, Jan. 8, 2002.
68. von Jezierski, Dieter, Slide Rules - A Journey Through Three Centuries, Translated from the German by Rodger Shepherd, Astragal Press, 2000, 126p.
69. von Jezierski, Dieter, "Two Tavernier-Gravet Slide Rule Systems," The Journal of the Oughtred Society, v.11, n.1, Spring, 2002, p. p.59-62.
70. Weinstock, D., "The RHS Calculator", The Journal of the Oughtred Society, v.6, n.2, Fall, 1997, p. 11-12.

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8. Logaritmal gridiron slide rule, from Internet site of Reinhard Atzbach, http://rover.vistecprivat.de/~ratz/rechsamm/ logaritmal.htm,
9. Goodchild tallying rule - author's collection.
10. Dumesnil, the "Régle Universelle Déposée - author's collection
11. Dempster RotaRule - author's collection
12. Gilson Atlas (21-cm dia disk) - author's collection.
13. Ross Precision Computer - author's collection
14. Appoullot Logz - author's collection
15. Alro Commercial slide rule - author's collection
16. Lafay Helice a Calcul - author's collection
17. Troeger 7.37-m rechenwalze - from Werner H Schmidt's Internet site: http://www.uni- greifswald.de/~wwwmathe/RTS/rs010.html.
18. 1.5-meter cylindrical slide rule (unknown maker) - author's collection

Table 1:

## Long Scale Slide Rules with Straight

## Formats

| Maker | Model | Configuration | Approx. <br> Date | Effective <br> Scale <br> Length cm | Data Source |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SINGLE SCALES (scale length $>50-\mathrm{cm}$ ) | Segments |  |  |  |  |
| Delamoriniere | Regle a calcul - 1.2m | 1 | 1863 | c50 to 60 | Hopp [26 ] \& Cajori [9] |
| Scofield | Engineer's Slide Rule | 1 | c1905 | 60 | Schure [55] |
| W. \& S. Jones (also Adams) | Robertson slide rule (sliding Gunter) | 1 | c1775 | c72 | Otnes[45] \& Cajori[9] |
| Ward, John | 2 to 3 ft . long slide rules | 1 | c1707 | 61-91 | Cajori[10] |
| Nestler | Reitz model \#24R | 1 | c1937 | 100 | Hopp[26] |
| Delaveleye | Regle a calcul - 2.3 m | 1 | 1863 | c115 | Hopp[26] |
| Lambert, Johan | 4 ft . scales | 1 | 1770s | c130 | Cajori[10] |

SEGMENTED SCALES Segments

| Favor, Ruhl \& Co. | M -D 7" Dual 10-5 | 2 | 1940s | 25 | my collection |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Faber-Castell | Novo-Duplex \#62/83 \& 63/83N | 2 | 1960s | 25 | von Jezierski [68] |
| Roos | Dual 10", 5". | 2 | 1940s | 25 | my collection |
| Nestler | Precision \#27/1 | 2 | c1915 | 30 | von Jezierski [68] |
| Tavernier-Gravet | $50-\mathrm{cm}$ long scale on $25-\mathrm{cm}$ slide rule | 2 | c1870s | 50 | Manley [39] |
| Nestler | Prazision No. 27 \& 0270 | 2 | c1910 | 50 | von Jezierski [68] |
| Dietzgen | Log Log Decimal Trig \#1741 | 2 | 1960's | 50 | my collection |
| Faber-Castell | Columbus \#3/42, \#342, \#3/42/342 | 2 | 1930's | 50 | von Jezierski [68] |
| Faber-Castell | Novo-Duplex \#2-83 \& \#2-83N | 2 | 1960's | 50 | von Jezierski [68] |
| Unique | 10/20 Precision | 2 | 1960's | 51 | Hopp[26] |
| Unique | Dualistic High-Speed | 2 | 1960s | 51 | Hopp[26] |


| Nestler | Precision \#27/9 \& \#27/a | 2 | ca. 1915 | 100 | von Jezierski [68] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Long | The 'Long' slide rule | 2 | ca. 1910 | 100 | Pickworth [47] |
| Post | Post Ritow Manifold \#1466? | 2 | 1925-1931 | 100 | my collection |
| Tavernier-Gravet | Lallemand | 2 | late 1800s | 100 | von Jezierski [69] |
| Whichmann | 100-cm long scale on $50-\mathrm{cm}$ slide rule | 2 | c1960s | 100 | Bready [8] |
| Unique | Pioneer Long Scale | 4 | 1960s | 112 | Barnes [6] |
| Anderson | Improved Slide rule | 4 | 1910 to 1920 | 120 | my collection |
| Hemmi | \#201 | 4 | 1960s | 200 | Feazel [19] |
| Hemmi | \#200 | 6 | 1930s | 240 | von Jezierski [68] |
| Hassler | Geodedic slide rule | 10 @ 2-cycle | c1850 | 305 | NIST [41] \& Cajori [9] |
| Nicholson | segmented slide rule scale | 10 @ 2-cycle | 1787 | 305 | Cajori[9] |
| Horner | segmented slide rule scale? | several | c1650 | ? | Cajori[9] |
| Scherer | Logarithmisch-graphische Rechentafel | ? | c1860s | ? | Cajori[9] |
| Péraux, E. | Échelle Logarithmique | 2 | c1860s | ? | Cajori[9] |
| Mannheim | side-by-side segmented scales | 2 | c1850s | ? | von Jezierski [68] |

GRIDIRON SCALES
Segments

| Billeter | Rechentafel - 0.5m | 4 | c1890 | 50 | Joss [34] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Billeter | Rechentafel - 1m | 8 | c1890 | 100 | Joss[34] |
| Aston \& Mander | Hannyngton's - small Extended (60") | 5 | c1880s | 159 | DeCesaris [14] |
| Jelinek | Logaritmal | 10 | c1943 | 150 | Atzbach [3] |
| Gilson | Pocket Slide Rule | 14 | c1915 | 178 | Richardson \& Clark [49] |
| Marotti | Lagartabla Gridiron | 14 | 1900s | c250 | IM2000 [28] |
| Cooper | 100-Inch | 20 | c1900 | 254 | Bennett [7] |
| Nicholson | long scale | 10 | 1797 | 305 | Cajori [9] |
| Aston \& Mander | Hannyngton's - large Extended (120") | 8 | 1880s | 318 | DeCesaris [14] |
| Billeter | Rechentafel - 4m | 10 | c1890 | 400 | Joss[34] |
| Cherry's | Calculator | 20 | 1880 | 508 | Pickworth [47] \& Cajori [9] |
| Calculigraph | Australian Slide Rule | 22 | c1909 | 549 | Petri-Palmedo [46] |
| Billeter | Rechentafel - 8m | ? | 1887 | 800 | Joss[34] |


| Gladstone's | Cross Gauge (like Everett gridiron) | $?$ | 1923 | 10.8-m |  |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Clairault, Jean | gridiron = 1500 French ft. | $?$ | 1720 | $500-\mathrm{m} ?$ |  |
| Scherer | Rechentafel | $?$ | 1892 | $?$ |  |
| Proell | Proell's Rechentafel | 20 | 1901 | $?$ |  |
| Everett | Universal Proportion Table Gridiron | $?$ | 1866 | Cajori [9] |  |
| Evans, Dr. J.D. | gridiron scale | $?$ | 1866 | $?$ | $?$ |

## SAW TOOTH SCALES

Segments

| Lurie | Precision | 1 | ca. 1910 | 50.8 | Babcock [4] |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Richardson | Pyramid \#1898 | 20 | ca. 1915 | 508 | Babcock [4] |

## CHARTS / TABLES / Segments GUNTER's LINES

| Merrifield | 24-inch Gunter Rule | 1 | 1850s | 30.5 | my collection |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Donn | Improved 'Gunter' Navigation Scale | 1 | c1850 | 61 |  |
| Forster | Gunter Rule ( 6ft. long double scale) | 1 | 1632 | 91.4 |  |
| C. Dumesnil | Graphic Table Rule | 8 | c1900 | Callection |  |
| MacMillan | Table Slide Rule | 20 | 1925 to 1930's | na |  |
| Loewe | Rechenscalen | 50 | 1893 | my collection |  |
| Goodchild | Mathematical Chart | 100 | c1902 | Ballantine [5] |  |
| LaCroix and Ragot | Graphic Table book | 1000 | c1930 | Holland [24] |  |
| Lalanne | Tableau Graphique | $?$ | c1846 | 111-m | $?$ |
| Tichy | Graphische Logarithment-Tafeln | $?$ | 1897 | La Croix \& Ragot [37] |  |
| Knowles | Calculating scale | $?$ | c1903 | $?$ |  |

Table 2:
Long Scale Slide Rules with Circular,

## Cylindrical and Band or Tape Formats



## CIRCULAR with CIRCULAR SCALES

| Nystrom | Nystrom's Calculator | 1 | c1850 | 36 | Miller [40] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Norma | Grafia 190 Rechenschiebe | 1 | c1970 | 50 | van Herwijnen [67] |
| Russian | CTM Military | 1 | 1970's | 55 | van Herwijnen [67] |
| Scott, Benjamin | 18" dia. circular slide rule | 1 | 1733 | 58 | Cajori [9] |
| Gilson | Binary \& Atlas | 1 | 1930s to 1960s | 64 | Aldinger \& Chamberlain [1] |
| Fuller/Palmer | Computing Scale | 1 | 1844 to 1870s | 67 | Feazel [18] |
| Troeger | nr. 1 ( 37/393/6004) | 1 | c1930s | 72 | Troeger [66] |
| East German | Military | 1 | c1970s | 75 | van Herwijnen [67] |
| Loga | 75 T (disk about 12 in dia.) | 1 | c1950s | 75 | Joss[32] |
| Oughtred (Elias Allen) | 12.5" dia. disk | 1 | c1632 | 76 | JOS[29] |
| Gilson | 'square' Atlas | 1 | 1920s | 78 | Aldinger \& Chamberlain [1] |
| Troger | Nr.1a (for persons with poor eyesight) | 1 | c1930s | 100 | my collection |
| Scott | 18" diameter circular slide rule | 1 | 1723 \& 1733 | c140 | Hopp[26] |
| Clairault, J.B. | 21" cardboard circular slide rule | 1 | 1727 | c150 | Hopp[26] |

CIRCULAR with CONCENTRIC CIRCULAR SCALES
Circles

| The Scientific Pub. Co. \& others | The Mechanical Engineer | 2 | c1900 | 17 | DeCesaris [15] |
| :--- | :--- | :---: | :---: | :---: | :---: |

| unknown Swiss company | The Mechanical Engineer | 2 | c1900 | 24 | DeCesaris [15] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Henri Chatelain \& others | Boucher (Calculigraphe) | 3 | 1870s to 1920s | 30 | my collection \& Hopp [27] |
| K\&E | Sperry | 3 (hybrid) | 1904 to 1940 | 32 | Otnes [43] |
| Fowler | Long Scale | 6 | 1920s to `1970s | 76 | Hopp [27] |
| Delamain | 2-disk with concentric circle long scales | 4 | 1632 | c112-140 | Cajori [9] |
| Pickett | 110 (copy of RotaRule) | 4 (hybrid) | 1960's | 127 | Shepherd [60] |
| Dempster, J.R. | RotaRule - Model AA | 4 (hybrid) | 1930's | 127 | Shepherd [60] |
| Boykin Products | copy of RotaRule | 4 (hybrid) | 1950's | 127 | Stanley [63] |
| Unique | Dial Calculator | 5 | 1930s ? | 127 | Hopp[26] |
| Fowler | Long Scale Magnum | 6 | 1940s to 1970s | 127 | Hopp [27] |
| Fowler | Jubilee Mag. Extra Long Scale | 11 | 1940s to 1970s | 185 | Hopp [27] |
| Sexton's | Omnimetre \#6 (Companion) | 20 | c1900 | 411 | Alteneder [2] |
| Biler | concentric semicircle scale | ? | 1696 | ? | Cajori [9] |

## CIRCULAR with SPIRAL

| Appoullot | Loga T3 \& T4 | 2 | 1920's | 68 | Schuitema [54] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Logomat | 816 (V \& G) | 3 | ca. 1970 | 110 | Schuitema [53] |
| Facini | spiral | 4 | ca. 1714 | 122 | JOS [30] |
| ALRO | 1010 Commercial | 6 | ca. 1940 | 150 | Schuitema [52] |
| Concise | M.V. Douglas | 10 | 1960's | 152 | O'Leary [42] |
| John Brown | spiral slide rule | 5 | c1660 | 213 | Science Museum [56] |
| Sutton (Oughtred) | none | 5 | 1663 | 280 | Science Museum [57] |
| Dixon | Combined Spiral Multi-Index Slide Rule | 10 | 1882 | 421 | Science Museum [58] |
| Adams | 12" dia. spiral sr w/10 windings | 10 | 1748 | c500 | Hopp[26] |
| Ross | Precision Computer | 25 | 1920's | 914 | my collection |
| Gilson | Atlas - Type III | 25 | 1930's | 1067 | Aldinger \& Chamberlain [1] |
| Gilson | Atlas - Type II | 30 | 1931 | 1186 | Aldinger \& Chamberlain [1] |
| Nicholoson | spiral | 10 | c1797 | 1250 | Cajori [9] |
| Gilson | Atlas - Type I (square | 30 | 1920's | 1400 | Aldinger \& Chamberlain [1] |
| Brown, John | spiral slide rule | 10 | c1660 | ? | Hopp[26] |
| Brown, John | spiral slide rule | 20 | c1660 | ? | Hopp[26] |


| Brown, Thomas | spiral slide rule | $?$ | 1631 | $?$ | Hopp[26] |
| :--- | :--- | :--- | :--- | :---: | :---: |
| Fearnly | Universal Calculator | $?$ | c1900 | $?$ |  |
| Milburn, William | spiral slide rule | $?$ | c1650 | $?$ | Cajori [9] \& Hopp [26] |
| Schuermann | Calculating Instrument | $?$ | c1896 | $?$ |  |

## CYLINDRICAL with HELIX

SCALES
Revolutions

| J.H. Steward | R.H.Smith Calculator | ? | c1906-1915 | 102 | Weinstock [70] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| J.H. Steward | R.H.Smith Calculator | 20 | c1906-1915 | 127 | Pickworth [47] |
| Otis King | Model "L" | 20 | 1920's to 1970's | 168 | Hopp [25] |
| Otis King | Model "K" (w/ log scale) | 20 | 1920's to 1970's | 168 | Hopp [25] |
| LaFay | Helice a Calcul \#1 | 50 | 1930's? | 254 | my collection |
| Stanley | 'Midget' (200") | ? | c1879 | 508 | Feely \& Schure [22] |
| Stanley | Fuller Model \#1 | 20 | 1878 to 1960 | 1270 | my collection |
| Stanley | Fuller Model \#2A (w/ sine scales) | 20 | 1878 to 1960 | 1270 | Hopp[26] |
| Stanley | Fuller Model \#2B (w/ log \& sine scales) | 20 | 1878 to 1960 | 1270 | Hopp[26] |
| Stanley | Fuller Model \#3 - Bakewell Stadia | 20 | 1878 to 1960 | 1270 | Hopp[26] |
| Dobie (Austrialia) | Collins Brown tubular calculator | 20 | c1960 | 1270 | Hopp[26] |
| Post | Model \#1475 Ritow Cylindrical | 20 | 1925 to 1927 | 1270? | Ross \& Hume [50] |
| MacFarlane | Cylindrical slide rule | ? | 1842 | ? | Hopp[26] |
| Mannheim | Régle á Calcul Cylindrique | ? | 1854 | ? | Hopp[26] |
| Mannheim | Régle á Calcul Cylindrique (wooden) | ? | 1871 | ? | Hopp[26] |
| Mannheim | Régle á Calcul Cylindrique (metal) | ? | 1873 | ? | Hopp[26] |
| Schukarev | Cylindrical slide rule | >80 ? | 1910 | ? | Davie [13] |
| Everett | Cylindrical slide rule | ? | 1866 | ? | Hopp[26] |
| Darwin | Spiral slide rule | ? | c1875 | ? | Cajori[9] |

## CYLINDRICAL with SEGMENT

Segments

| Daemen-Schmid / Loga | Loga Cylindrical 1.2 m | 20 | started in 1896 | 120 |  |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Trogger | n. 4 - small format (5 Bavarian feet ) | $10 / 20$ | c1920s | 147 | Joss [33] |
| no name (Troeger clone?) | Cylindrical slide rule 1.5m | $10 / 20$ | c1920s | 147 |  |


| Nestler | Cylindrical Calculator 1.6m | 16 | 1922-1937 | 160 | Joss [33] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| no name | Cylindrical slide rule 1.6m | 16 | ? | 160 | Joss [33] |
| Billeter | Cylindrical slide rule | 16 | started in 1888 | 200 | Joss [33] |
| Daemen-Schmid / Loga | Loga Cylindrical 2 m | 20 | started in 1896 | 200 | Joss [33] |
| Schneider, Edmund - Munich | Reciloga 2m | 15 | ? | 200 | Joss [33] |
| Daemen-Schmid / Loga | Loga Rechenwalz 2.4 m | 20 | started in 1896 | 240 | Joss [33] |
| Nestler | Cylindrical Calculator | ? | 1922-1937 | 375 | von Jezierski [68] |
| Muto Giken | Kooler Calculator | 50 | 1960's | 491 | Shepherd [59] |
| Trogger | nr. 3 - large format | 32 / 64 | patented 1908 | 737 | Troeger [66] |
| no name (Troeger clone?) | Cylindrical slide rule 7.37m | $32 / 64$ | c1930s? | 737 | Joss [33] |
| Daemen-Schmid / Loga | Loga Rechenwalz 7.5 m | 40 | started in 1896 | 750 | Joss [33] |
| National | Cylindrical slide rule 8m | 40 | started in 1916 | 800 | Joss [33] |
| Thacher | Cylindrical Slide Rule | 40 | 1883 to 1940's | 914 | Feely \& Schure [21] |
| Billeter | Cylindrical slide rule | ? | started in 1888 | 1000 | Joss [33] |
| Daemen-Schmid / Loga | Loga Rechenwalz 10m | 50 | started in 1896 | 1000 | Joss [33] |
| National | Cylindrical slide rule 10 m | 50 | started in 1916 | 1000 | Joss [33] |
| Daemen-Schmid / Loga | Universal 12m | ? | started in 1896 | 1200 | Joss [33] |
| Nestler | Ronda III | ? | 1922-1937 | 1250 | Hopp [26] \& von Jezierski [68] |
| Numa | Cylindrical slide rule 14.4m | 50 | ? | 1440 | Joss [33] |
| Daemen-Schmid/Loga | Loga 15-meter | 60 | started in 1896 | 1500 | Joss [33] |
| Billeter | Cylindrical slide rule 16m | ? | started in 1888 | 1600 | Joss [33] |
| National | Cylindrical slide rule 16 m | 80 | started in 1916 | 1600 | Joss [33] |
| Daemen-Schmid | Loga Cylindrical 24 m | 80 | started in 1896 | 2400 | Joss [33] |
| Reiss | Cylindrical | ? | c1950s ? | ? | Hopp [26] |
| MacFarlane | Cylindrical (scale type uncertain) | ? | c1842 | ? | Cajori [9] |
| Hoyau | Boîtes á calculer ( type uncertain) | ? | c1816 | ? | Cajori [9] |

CYLINDRICAL with RING SCALES

Rings

| Delamain | Great Cylinder - 1 yard diameter | 10 or more | c1632 | c30-m | Cajori [10] |
| :--- | :--- | :---: | :---: | :---: | :---: |

TAPE and BAND SLIDE
RULES

## Bands

| Paisley | Calculator, Model A (band slide rule) | 2 | c 1939 | 50.8 | Feely [20] |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Wichmann | Measuring tape slide rule | $?$ | c 1938 | 50.0 | Hopp [26] |
| Darwin | Metalic Tapes | $?$ | 1875 | $?$ | Cajori [9] |
| Tower | Metalic Tapes | $?$ | 1885 | $?$ | Cajori [9] |
| Silvio Masera | Rechenstab mit Endlosband | $?$ | 1902 patent | $?$ | Joss [32] |

## The Abacus: one of the oldest Calculation Devices

## Jörn Lütjens

## Some information on historical facts, systems, special constructions and calculation examples

## History of the abacus in brief

The abacus was one of the earliest, and most effective calculating devices. In the hands of a skilled operator it was a powerful aid to computation. In several mathematic-historically literature $[3,4,5,8]$ you can find out that the definite origin of the abacus is obscure. But there is some reason for believing that its earliest form was a reckoning table covered with sand or fine dust, in which figures were drawn with a stylus, to be erased with the finger when necessary. This description is the meaning of the Greek word abax (dust) and from that, it seems the abacus may have etymologically derived. As you know today we use the Latin word abacus. Various forms of a line abacus were in common use in Europe until the opening of the seventeenth century. A third form of abacus appeared in certain parts of the world: instead of lines the table had movable counters sliding up and down grooves. All three types of abacuses were found in ancient Rome - the dust abacus, the line abacus, and the grooved abacus. The rows represented units, such as tens, hundreds, and the quantity of counters in the rows represented a number.


Reproduction of a Roman abacus [2]

The abacus became rapidly indispensable, mainly because of its simplicity of use in societies where number systems were complex and difficult for reckoning. This was the case with Roman numerals (try, for example, dividing CCCXXXXXXIII by XXXIII in your head) [it is $363: 33=11$ ]. By the end of the sixteenth century, the modern number system based on Hindu-Arabic mathematics had become widespread. There is a sixth-century Chinese reference to an abacus on which counters were rolled in grooves. It seems to be by far the earliest work which speaks of "ball arithmetic". The device mentioned above has one bead in an upper division and four beads in a lower part [9]. The description of this ancient Chinese abacus and the known intercourse between East and West give reason to believe, that the Chinese abacus was suggested by the Romans.

Increasing trade relations during the Han Dynasty 200 BC - 200 AC with the Roman Empire are indicators for this assumption [8]. The earliest printed Chinese document, showing an abacus is from 1593 and has led many historians to conclude that the abacus did not become known in China until the end of the sixteenth century. Nevertheless it is not sure that the Roman abacus was the direct ancestor of the form of abacus, which is still used in East-Asia (China, Korea, Japan). Referring to Chinese and Japanese Historians there are some indications that the development of an abacus occurs in the East (India, China) and in the West independently nearly at the same time [6]. Unfortunately the results of studying this literature are confusing.

The present Chinese bead abacus, which is called suanpan in Mandarin and soopan in the southern dialect, was a later development, probably appearing in the twelfth century, and did


An early printed Chinese picture of the abacus, from the Suan Fa Thung Tsung, A.D. 1593 [9]
not come into common use till the fourteenth century. The Japanese are said to have obtained a form of abacus from China, through Korea, in about the sixteenth century.

The Korean word for abacus is tschu pan and the Japanese word is soroban. Both words are probably renderings of the Chinese suanpan. In spite of different languages, at least they sound almost similar.
In Japan the form and operational methods of the abacus have undergone some improvements. Like the present-day Chinese suanpan, the soroban long time had two beads above the beam and five below. But toward the close of the $19^{\text {th }}$ century it was simplified by reducing the two beads above the beam to one, and finally, around 1920 to 1930 it acquired its present shape by omitting yet another bead, reducing those below the beam from five to four [6]. Thus the present form of the soroban looks more elegant, is much smaller and sometimes it has more columns, as many as thirty-one. But those with fewer columns ( 13 or 15 ) are mostly used. But there is one interesting question:

Why does the modern Japanese soroban have only 5 beads on each rod, limited to display the number 9 ? And why does the actual Chinese suanpan have 7 beads on each rod, which can display in total the number 15 ? What are the advantages and disadvantages?

But no matter which version a Chinese or a Japanese, with the bead or rod abacus for many centuries calculations could be done much more quickly than on paper. This typical Asian speciality has come to end with the spreading of electronic calculators. But a final rear up against the new technology happened when the abacus won over the electric calculator in an exciting contest in 1946 in Tokyo [5].

## Constructions of different kinds of wide spread abacuses

A crosswise bar divides the abacus into two parts. The upper part is called "heaven" and consists of five-value beads. The lower part is called "earth" and consists of one-value beads.

It is the zero-position of all abacuses, if the upper beads are moved up and the lower beads are moved down. Only if one or more beads are moved toward the beam, they represent a number.

In principle the rods represent (starting from the right side and continuing to the left side) the "ones", the "tens", the "hundreds" etc. The user has to decide which rod the "ones" should be. If there are numbers with values less than one, it is convenient to put the column with "ones" just in the middle of the abacus. So numbers can grow to both sides. But if there are only numbers with values from one and higher you can start just at the right side of the frame developing the calculation in direction of the left side.


Japanese soroban with $(5+1)$ beads on each rod.


Japanese soroban with (4+1) beads on each rod.

## Some details of construction:

|  | Chinese | Japanese, Korean |
| :---: | :---: | :---: |
| Frame <br> made of: colour: | wood, plastic black, brown | wood, plastic black, brown |
| Beads <br> material: colour: shape: | wood, plastic <br> black, red round | wood, plastic <br> different colours, but mostly brown double coned |
| Rods and beads | relatively free motion between the beads and rods. | close-fit sliding between beads and rods. |

In addition to the East-Asian types there is a third form, the Russian abacus that is called stschoty. It is not sure whether the stschoty is derived from the suanpan. But anyway (direct or indirect) the stschoty has been in Russia the only calculation device for many centuries up to now. It is surprising that the dimensions of a standard office stschoty measure about 28 cm width by 46 cm height. Smaller stschotys, mainly for students are also in use. They

have a size of about 13 cm width by 18 cm height. The main differences to the Far East abacuses are as follows:

- The iron made rods are lying in a horizontal position and the beads have to move from the right to the left of the frame.
- The stschoty doesn't have a separate section for making groups of higher values.
- The frame has 10 to 13 rods with each 10 beads (depends on the size of the stschoty).
- One or two rods are equipped with only 4 beads. These columns are for calculations with fractions (1/4 Roubles or 1/4 Kopeks).

As you see the structure is a very simple one. In the picture the fourth rod (counted from the bottom) represents the "ones". The next above stands for the "tens" and so on. The uppermost rod represents the "millions". Otherwise the rod with only 4 beads has another function too: it indicates the position of the comma
Russian stschoty and the next rod just below represents the "tenths" and the next below is for the "hundredths".

Referring to Menninger [8] the form of the Russian abacus spread out to France in 1812 and later came into common use in Germany as a type of children's calculator with 10 beads each on 10 rods. Up to now it is still in use for teaching elementary mathematic - and in other places of the world too, especially as a "Number Aid" or "Ball Abacus" in the USA.

## Calculation with suanpan and soroban: Addition $54+46$

## Chinese system


placing 54:
move up the four lower beads in the first column. Move down one upper bead in the second column. The position shows 54
adding 46:
Move down one upper bead in the first column and move up one lower bead in the first column. Move up four lower beads in the second column. The positions of all beads together display the result of one 100 .
redundant bead positions need clearing: cancel the two upper 5 beads of the first and second column and move down all lower beads from the first and second column. Move up one lower bead from the third column, which represents the result of one 100 with only one bead.

## Japanese system

placing 54:
move up the four lower beads in the first column. Move down the upper bead in the second column. The position shows 54
adding 46 :
40 can be added but not the 6 . Because of not enough beads use the supplementary number $50-4$ ! But in the second column 50 cannot be added directly.

adding 50:
cancel the upper bead in the second column and move up one bead in the third column ("hundreds").
The result is now 104

subtract 4:
after moving down the four beads in the "ones"-column the final result is one 100

A skilled user of the suanpan is able to make the step after adding 46 directly to the "one-hundred-bead" at the third rod. But if you are not very familiar with the speedy procedure, you can work step by step with the suanpan. The following pictures show that after adding 46 three more intermediate steps have to be executed.

after adding 46,
1st step of clearing: cancel all five lower beads and move down the second " 5 bead". They together also represent the result one 100 .


2nd step of clearing: cancel the two upper 5 beads and move up one lower bead from the second column, which represents the


3rd step of clearing: cancel all five lower beads of the second column and move down the second " 5 bead".

This example shows that the Chinese suanpan with (5+2) beads can display the same number in different ways. But skilled users don't perform calculations in the way demonstrated above. The method of thinking by operating an abacus is important. The user normally thinks of the next higher power to ten: e.g. 9 are $10-1$ or 17 are 20-3 or 98 is $100-2$ and so on. Mental calculation is different from pencil arithmetic, which is performed in thinking of the total.

But the abacus' users' mental calculations are limited to a maximum of ten and each step is performed independently without being concerned with the total. On the one hand, in case of the Japanese soroban is to consider that because of the reduced number of beads this mentioned principle of mental calculation is much more important than for the Chinese suanpan. On the other hand the Chinese suanpan with two five-value beads gives advantage for division calculating to indicate in one column a higher number of 10 .

## Special abacuses

## Lee's Improved Abacus

invented around 1950 by Lee Kai Chen (Taiwan) [7]


## Construction detail:

Place setting vernier: a white movable rubber string with red dot markers makes it easier to fix the positions for the "ones", "tens", "hundreds" and so on.

This double abacus has a size of 33 cm by 20 cm . It combines a traditional Chinese suanpan ( $5+2$ ) with a Japanese soroban $(4+1)$. The lower part of the suanpan with the big beads is the main abacus. This section is called "Principle

Operation Filed" P-O. The upper abacus containing small beads is an auxiliary abacus. This section is split in a left and right part - it is called "Auxiliary Operation Fields" (left A-O and right A-O).

Lee Kai Chen praises as an important merit of his improved abacus the combination as well as the partition of the three counters. Placing different calculations makes more complex processes visible comparable to an old-style abacus.

Lee Kai Chen praises as an important merit of his improved abacus the combination as well as the partition of the three counters. Placing different calculations makes more complex processes visible comparable to an old-style abacus.

## Extraction of square roots with the Lee's abacus

At first: remember, the method of extracting square roots we learned in school is based on the term as the

| $\sqrt{841}=$ | result: 29 |
| :--- | :--- |
| 4 | $: 2$ |
| $\overline{441}$ | $:(4+9) 9$ |
| 441 |  |
| $\overline{0}$ |  |

With the aid of an abacus the extraction method is turned into a process of additions and subtractions. This method looks very easy to handle, but it is somewhat slow and tedious. Especially in case of long square root numbers a series of additions and subtractions will take place.


Place the number 841 from which the square root is to be extracted in the centre of the P-O field.
Divide the square number into two groups, the first group containing one figure, 8 and the other group containing two figures, 4 and 1 .

Move the indicator to mark out the first group
step 1


Place 1 on the first column of the right A-O field. This is now the "root" number.
Subtract 1 from the figure of the first "square" group in the P-O field, namely 8 , leaving 7 .

step 3


Add 2 to the root number 1 making 3 and subtract the sum 3 from the first group in the P-O field. There is now 4.

Add 0 to the 3 in the "root" number in the A-O field. Now there are two columns to be considered. To the value of 30 add 11. The sum of the new "root" number is 41 .

Move the indicator in the P-O field two columns to the right. In the square number are now 441.

To avoid a boring repetition of pictures with certain number constellations, all 11 steps are shown in the following table: [7]


Here are the last final steps (No 9, 10 and 11) at the abacus:


Add 2 to the root number 53 making 55 and
subtract the sum 55 from the square number in the P-O field, leaving 57.


Add 2 to the root number 55 making 57 and subtract the sum 57 from the square number in the $\mathrm{P}-\mathrm{O}$ field, leaving zero.
step 11


Add 1 to the "root" number, making 58. Divide 58 by 2 ; the quotient 29 thus obtained is the square root required.

As we see, the procedure is not more than adding a number to the "root" number and subtracting the sum from the "square" number until the original number is exhausted.

An abacus with double-five beads on each rod
(Usable for a quinary numerical system? More questions than answers)


This abacus from Vietnam has in the lower and in the upper section 5 beads each on every rod. It measures 160 mm by 90 mm .

What kind of concept or calculation system is behind this unusual placement of beads?

## First supposition:

The 10 beads in total on each rod indicate, that this abacus is usable in the same way like the Russian stschoty, based on the decimal system. From the right to the left each column represents a power to 10. If so, then the horizontal beam, separating the abacus into two sections, has no special function.

## Second supposition (if the upper field is not in use):

If we switch to a quinary system (base 5), than the first rod represents the "ones" $5^{0}$.
The next rod stands for the "fifth" $5^{1}$, then follow the "twenty-fifth" $5^{2}$, the "hundredtwenty-fifth" $5^{3}$, and so on following by the other powers to 5 . On each rod we can count 10 beads with the same value of $5^{\mathrm{x}}$. But in principle all beads (counting from 6 to 10) in the upper section are redundant because the same number can displayed by transformation to the rod next left. The column has to be cleared (this is the same procedure like the suanpan with $5+2$ beads). In this case the horizontal beam has no special function either and the upper five beads on each rod are unnecessary.

## Third supposition:

With integration of the upper field it may be usable for a combination of decimal and quinary system.


Transformation from the quinary
to the decimal system:


But the basic question remains: Is it possible that this abacus from Vietnam is used for a quinary system? On the other hand a Chinese suanpan shows the same characteristic if we focus on the lower section with 5 beads only. What purpose do the upper 5 beads have? The idea of a quinary system is tempting of course but may be absolutely wrong.

I've tried to follow another idea: a possible correlation between finger counting and use of an abacus. Of course counting systems and calculation systems are not the same. In literature I found a reference of using the rare base 5 system. Georges Ifrah mentioned that today in the Bombay region of India some merchants still use an interesting counting technique with fingers. In addition, I was a witness of using another five numerical system by fingers in South Korea. Here some basic information:

Using fingers: the natural base for a quinary system

one
two
three
four
five
five-and-one
five-and-two
five-and-three
five-and-four
two-five
two-five-and-one
two-five-and-two
two-five-and-three
two-five-and-four
three-five
three-five-and-one
three-five-and-two
three-five-and-three
three-five-and-four
four-five
four-five-and-one
Oral numeration:
In a quinary system there are independent names for the first five whole numbers and the powers of 5 . Other numbers are expressed by combinations of these names [4].

Today there are very few quinary systems existing in a pure state. In South Korea's harbour city Pusan I attended 1987 at a fish auction and detected bidders using a one hand quinary system without shouting their bids (as normally happened at auctions).


Wall poster with instructions in the auction hall


Korean bidder at a fish auction. Look at the jackets - they are used as shield against competitors.

It is not clear that there is a close relationship between a quinary numerical system (weather applied by oral numbers or shown by fingers) and the abacus with ( $5+5$ ) beads as shown above. But it is likely.

Recently I found a web site of a Malaysian abacus museum with a lot of abacus pictures. Two of them are also equipped with ( $5+5$ ) beads. Unfortunately there was no information available. [1].


## Conclusion

There is no doubt: For about 50 years the abacus has lost its importance as a calculation device for many centuries. In the view of most people today, an abacus is not more than a nice accessory of typical Chinese or Japanese culture. And because of its very simple construction some people think that it is not worthwhile to look behind the idea of mental arithmetic and primitive execution by moving beads. Simple things are always fascinating, especially if we consider that Asian people handle an abacus with excellent mastership. The development of different types of abacuses are proof enough. The author's intention was not only, to point out principles of abacus calculating with an focus on some special constructions.

## References

[1] Abacus Museum at the Malaysian web site of ALOHA Mental Arithmetic http://www.alohama.com/abmuseum/functional-abacus/index7.html
[2] Großes Handbuch der Mathematik, Köln 1968
[3] Hartmuth, Maximilian: Vom Abakus zum Rechenschieber. (Nachdruck der Ausgabe von 1942). Osnabrück; Otto Zeller, 1968
[4] Ifrah, Georges: From one to zero. English translation of "Histoire universelle des chiffres", Harrisonburg,Virginia, 1985. ISBN: 0-670-37395-8
[5] Kojima, Takashi: The Japanese Abacus - its use and theory. Rutland, vermont \& Tokyo 1954
[6] Kojima, Takashi: Advanced abacus. Japanese theory and practice. Rutland, Vermont \& Tokyo 1963
[7] Lee Kai-Chen: A Revolution of Chinese Calculators. How to learn Lee's abacus. Download on the website of Luis Fernandes http://www.ee.ryerson.ca:8080/\~elf/abacus/leeabacus/index.html\#cubert
[8] Menninger, Karl: Kulturgeschichte der Zahlen. Breslau 1934
[9] Ronan, C.A. / Needham, J.: The shorter science and civilisation in China II, Cambridge 1981/6

## The IM 2000+ Challenge Drum

## Leo de Haan

## International Meeting 2000 in the Netherlands

It was during an IM2000 preparatory meeting, three years back, that IJzebrand Schuitema again launched one of his many ideas: to create a banner or ensign bearing the dates and locations of all subsequent International Meetings.

As usual the idea was tossed around mercilessly until it gradually evolved into an attractive "challenge token", to be rotated between the national organisations which accept the yearly challenge of realising an International Meeting. Leo de Haan took on this idea, and designed a stylized model of a "drum-type" slide rule, turned and chiseled from raw wood, but very recognisable for the connoisseur as modeled after a LOGA "roller".

## Construction

The Challenge Drum is constructed of Merbau hardwood, the text engravings are on brass plates, and the other fittings are Volkern or brass.


IM 2000+ Challenge Drum

## Specifications:

- Drum:

Merbau, 258 mm long, main diameter 78 mm

- Nameplates: Brass, 123 mm long, 10 mm wide and 2.0 mm high
- Lettertype Helvetica, height 4mm, engraved in black
- Axles: $\quad$ Solid brass, diameter 6 mm
- Stands: Volkern, 6 mm thick, 78 mm high, 84 mm wide at base
- Holder tubes: Brass, diameter $10 \mathrm{~mm}, 1 \mathrm{~mm}$ thick
- Packing: Wooden box with sliding lid, external dimensions $403 \times 133 \times 123 \mathrm{~mm}$

Since IM2000 the Challenge Drum is officially handed over, with appropriate speeches at the end of the meeting, by the Organizing committee of the current IM to the Organising Committee of the next IM.

The drum provides space for 19 nameplates, of which nine have been used after IM2003: so we can continue engraving and challenging until we meet our own "year 2014" problem. Then we will have to meet the challenge to launch a new idea for the commemoration of our steady and successful flow of yearly IM's.

## Dinner Guest: Wim Granneman

## Guest during Dinner Party on Friday September 19, 2003.

The tulip has, from cultural point of view, taken up an important position in the Dutch history of art, as well as later in economical respect. The origin of the tulip lies in Asia, from China in the far east and more westward, growing in wild. Traveling clerics brought the bulbs into Turkey. And there it was cultivated and got much appreciation, especially by the well-to-do people. It developed to a kind of status symbol, connected with prosperity and wealth. And this got a reaction in art.

The bulb came from Turkey in Western Europe, and especially in the Netherlands. And here as well, people discovered its beauty, and again, like in Turkey, the possession of bulb and flower was considered as a witness of being a rich person. Soon it was translated artistically in two ways. For people who could not afford to pay much money for a real bulb or flower, countryside style artists painted cupboards, chests, etc. with tulip flowers. For those people it stood for a real flower. The second artistic translation were the famous flower still life's, bought by the wealthy citizens to decorate the rooms in their houses. And this also was considered as a real bouquet.

The Netherlands and Flanders are well known for the
 many still life painters. To buy bulbs and flowers cost fortunes and still life paintings were also very expensive. Many people launched into bulb speculations. But like in many such rages, the market sometimes suddenly collapses. So the tulip market did. Fortunes were lost. One might expect that the appeal would totally disappear. But not so with the tulip. The artistic translation came to an end but the cultivating of the tulip bulb was continued to be an attractive product for everybody. An intensive culture was started and a large turnover was the result, in the Netherlands as well on markets abroad. A huge market worldwide was opened.

One of the convenient circumstances was that the nature of the soil behind the dunes in the provinces of North- and South-Holland was exceedingly appropriate for cultivating tulips and other bulbs, and this resulted in a large area of flower bulb culture.

Consequently a bulb culture research was started, in laboratories as well as on experimental fields, to get a top quality product. The Netherlands scored high in this aspect and earned world reputation. Many specialists have been active in this field of research.
One of the most important persons of all of them is Mr. Wim Granneman. He studied many years the results of cultivating and did many laboratory tests. The results of his activities is laid down in a close analysis how to cultivate, to dry, to heat, to cool, etc. the bulb, finally to get the best product and the best flower, and predict the exact date the bulb will give its flower.

Mr. Granneman has translated his research results into a kind of calculating disc, called "Forcing Disc". Using this forcing disc one can calculate exactly how long cooling and warehousing has to be done to get a pre-appointed flowering result.

It has been by chance that Mr. Granneman got in contact with the Dutch "Kring". This resulted in an invitation to him to be as guest of honour, accompanied by his wife, at the dinner party on Friday September 19, and to give a short lecture on his research activities, resulting in his "Forcing Disc".

Considering the world fame of the Netherlands concerning flower bulb culture, combined with having acquaintance with the designer of the forcing disc that takes up such an important place in this specialty, the "Kring" has considered itself fortunate to introduce Mr. Granneman to the participants of IM2003 and to ask him to tell about his fascinating specialty.

It is in more than one aspect that this point of program will leave behind a lasting remembrance to all participants present at this dinner party !!


[^0]:    ${ }^{1}$ In 1543 Copernicus published a book in which he proclaims that the earth turns around the sun, instead of the opposite.
    ${ }^{2}$ Galileo, who mathematically proved Copernicus' statement had to recant this later on.

[^1]:    ${ }^{3}$ H. de la Fontaine: De zeventiende eeuw. Het Hollandse wonder, in: Boeken in Nederland. Vijfhonderd jaar schrijven, drukken en uitgeven, Grafisch Nederland 1971 p 46 e.v.
    ${ }^{4}$ De kleurrijke wereld van de VOC p 12

[^2]:    ${ }^{5}$ K.A. Ottenheym: Philips Vingboons p 169; J.J. Terwen en K.A. Ottenheym: Pieter Post p 217 e.v.

