

Negenproef

Doel: controle van berekening

$\begin{array}{r} 12 \\ \times 257 \\ \hline = 3084 \end{array}$	$1+2 = 3$ $2+5+7 = 14 \rightarrow 1+4 = 5$ $3+0+8+4 = 15 \rightarrow 1+5 = 6$ $3 \times 5 = 15 \rightarrow 1+5 = 6$	
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De wiskunde:

$a = \sum_{i=0} 10^i a_i = \sum_{i=0} (9 + 1)^i a_i$
 a_i is i^e cijfer van a

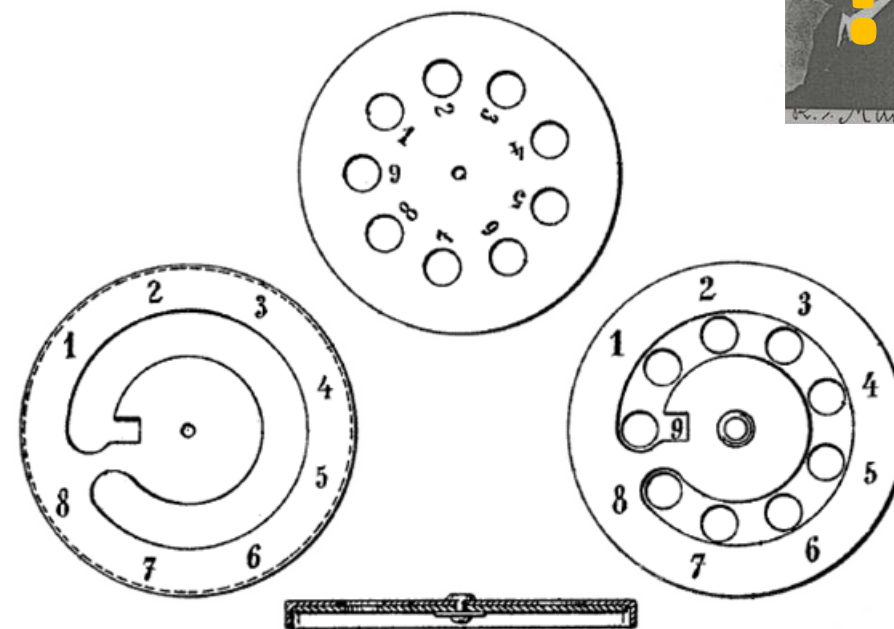
$a \bmod 9 = (\sum_{i=0} a_i) \bmod 9$

$a \circledast b = c \quad (a \bmod 9) \circledast (b \bmod 9) = c \bmod 9$

\circledast is + of - of \times of \div

Calculator:

Rudolf Malý, 1914



Elfproef

Doel: controle van berekening

$\begin{array}{r} 12 \\ \times 257 \\ \hline = 3084 \end{array}$	$\begin{aligned} -1+2 &= 1 \\ 2-5+7 &= 4 \\ -3+0-8+4 &= -7 \end{aligned}$
	$-7 \rightarrow 11-7 = 4$
	$1 \times 4 = 4$

☺

De wiskunde:

$i = 0$: eenheid, 2: honderdtal, ... $i = 1$: tiental, 3: duizendtal, ...

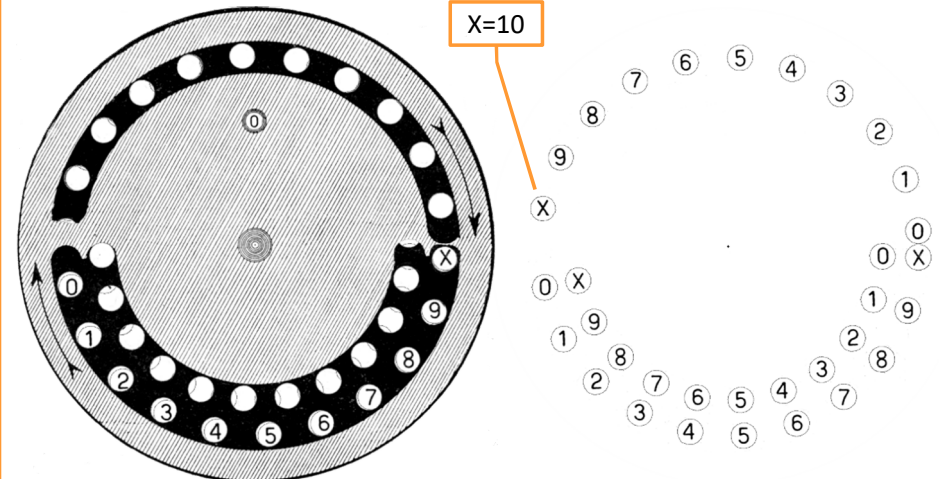
$$a \bmod 11 = \left(\sum_{i_{\text{even}}} a_i - \sum_{i_{\text{oneven}}} a_i \right) \bmod 11$$

\odot is + of - of \times of \div a_i is i^{e} cijfer van a

$$a \odot b = c \qquad (a \bmod 11) \odot (b \bmod 11) = c \bmod 11$$

Calculator:

Controlex
ReumHelm BV, 1953



Creditcard



Doel: controle van getal a

$$\left(\sum_{i=0}^{11} \begin{cases} i \text{ odd: } 2a_i \bmod 9 \\ i \text{ even: } a_i \end{cases} \right) \bmod 10 = 0$$

a_i is i^{e} cijfer van a

De wiskunde:

gewicht i^{e} cijfer van a

$$\left(\sum_i w_i a_i \right) \bmod p = 0$$

- 1 van p willekeurige fouten wordt gemist
- maar fouten zijn niet willekeurig!

Calculator:

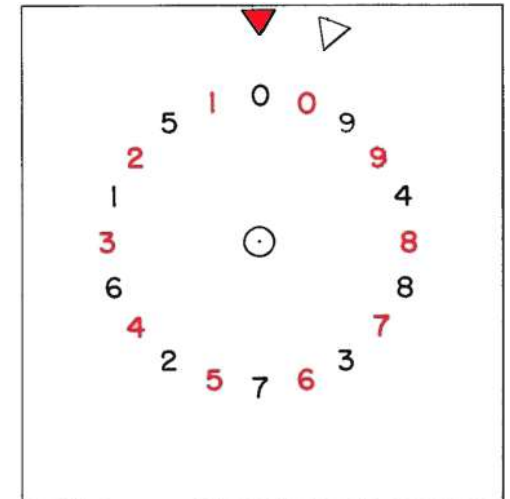
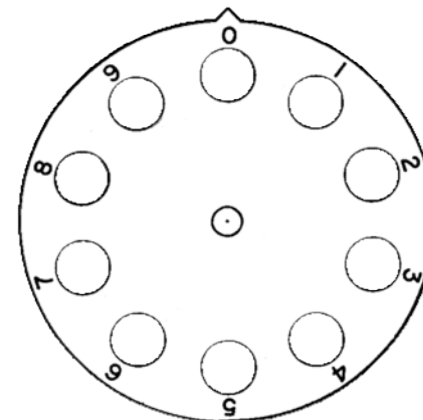
L.C. Zitnik (Pure Oil Comp.) 1959

Algoritme:

Hans P. Luhn (IBM) 1954



Hans P. Luhn



Verbeterde formule

Doel: controle van getal a

$$(3a_1 + 6a_2 + 8a_3 + 5a_4 + 7a_5 + 10a_6 + 2a_7 + 4a_8 + 9a_9 + a_{10}) \bmod 11 = 0$$

a_i is i^e cijfer van a

a_{10} is check digit

De wiskunde:

gewicht

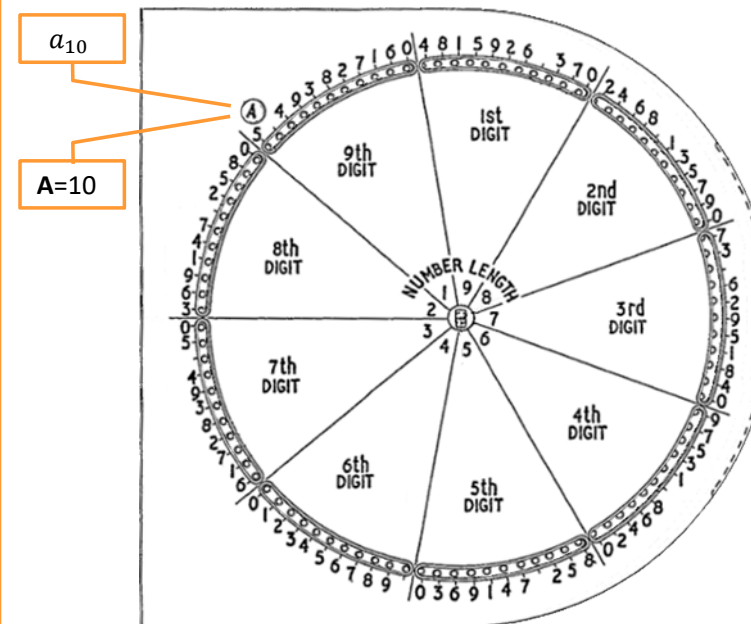
i^e cijfer van a

$$\left(\sum_i w_i a_i\right) \bmod p = 0$$

- verschillende gewichten voor elke positie i
- priemgetal p is beter!

Calculator:

A. H. F. Richardson, 1965



Verhoeff-algoritme



Doel: controle van getal a

$\sigma(x)$ is permutatie

a_i is i^e cijfer van a

$$\sigma(a_n) \otimes \sigma^2(a_{n-1}) \otimes \dots \otimes \sigma^n(a_1) \otimes a_0 = 0$$

\otimes : vermenigvuldiging in puntgroep D_5

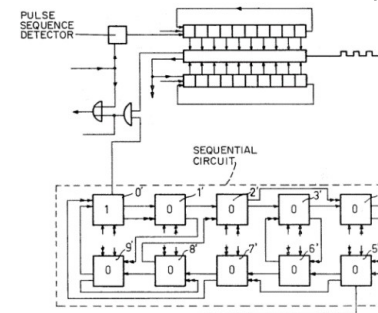
De wiskunde:

D_5	0	1	2	3	4	5	6	7	8	9
0	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆
1	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆
2	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆
3	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆
4	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆
5	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆
6	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆
7	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆
8	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆
9	☆	☆	☆	☆	☆	☆	☆	☆	☆	☆

x	0	1	2	3	4	5	6	7	8	9
σ	1	5	7	6	2	8	3	0	9	4
σ^2	5	8	0	3	7	9	6	1	4	2
σ^3	8	9	1	6	0	4	3	5	2	7
σ^4	9	4	5	3	1	2	6	8	7	0
σ^5	4	2	8	6	5	7	3	9	0	1
σ^6	2	7	9	3	8	0	6	4	1	5
σ^7	7	0	4	6	9	1	3	2	5	8
σ^8	0	1	2	3	4	5	6	7	8	9

Calculator:

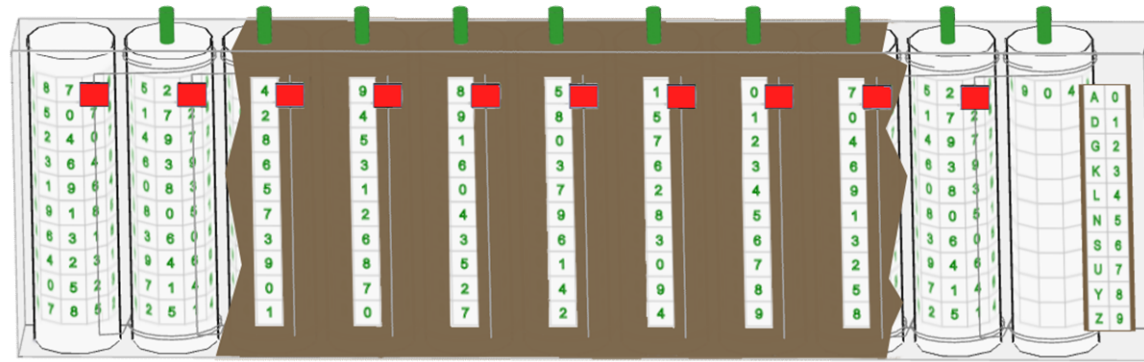
J. Verhoeff (Philips), 1969



Andries de Man, 2017

8	7	9	6	4	1	6	8	2	1	3	5	5	2	7	5	7	3	8	3	0	7	8	9	7	9	8	9	7	0	5	6	7	A	0					
5	0	7	0	6	4	5	6	8	9	1	3	3	5	6	5	7	4	8	3	6	7	0	7	9	3	9	5							D	1				
2	4	0	5	0	6	4	5		4	9	1	4	3	5	0	8	5	9	4	8	5	6	7	4	0	7	8	3	1						G	2			
3	6	4	1	5		2	4	5	5	4	9	7	4	3	3	0	8	0	9	4	8	8	5	6	6	4	0	2	8	4						K	3		
1	9	6	4	1	5	8	2	4	3	5	4	2	7	4	7	3	0	1	3	0	9	8	9	5	9	6	7	2	8							L	4		
9	1	8	7	9	3	0	9	3	0	7	8	1	6	0	9	2	4	1	5	7	2	3	4	1	8	5	4	6	2								N	5	
8	3	1	2	7	9	1	0	9	2	0	7	9	1	8	6	9		2	1	5	1	2	3	3	1	8	1	4									S	6	
4	2		6	2	7	7	1	0	6	2	0	8	9	1	1	6	9	6	2	1	0	1	2	2	3	1	5	1	3									U	7
0	5	2	3	8	2	3	7	1	8	6		0	8	9	4	1	8	7	6		4	0	1	5	2	3	0	5	9									Y	8
7	8	5	9	3	8	9	3	7	7	8	6	6	0	8	2	4	1	5	7	8	3	4	0	8	5	2	6	0	7									Z	9

Verhoeff



10=links	9	8	7	6	5
7 9 6 4 0 1 3 2 5 8	2 8 3 9 7 0 6 4 1 5	4 5 6 8 2 7 3 9 0 1	9 1 3 5 4 2 6 8 7 0	8 0 6 1 9 4 3 5 2 7	5 7 3 0 8 9 6 1 4 2
0 7 9 6 4 8 1 3 2 5	7 2 8 3 9 5 0 6 4 1	2 4 5 6 8 1 7 3 9 0	4 9 1 3 5 0 2 6 8 7	9 8 0 6 1 7 4 3 5 2	8 5 7 3 0 2 9 6 1 4
4 0 7 9 6 5 8 1 3 2	9 7 2 8 3 1 5 0 6 4	8 2 4 5 6 0 1 7 3 9	5 4 9 1 3 7 0 2 6 8	1 9 8 0 6 2 7 4 3 5	0 8 5 7 3 4 2 9 6 1
6 4 0 7 9 2 5 8 1 3	3 9 7 2 8 4 1 5 0 6	6 8 2 4 5 9 0 1 7 3	3 5 4 9 1 8 7 0 2 6	6 1 9 8 0 5 2 7 4 3	3 0 8 5 7 1 4 2 9 6
9 6 4 0 7 3 2 5 8 1	8 3 9 7 2 6 4 1 5 0	5 6 8 2 4 3 9 0 1 7	1 3 5 4 9 6 8 7 0 2	0 6 1 9 8 3 5 2 7 4	7 3 0 8 5 6 1 4 2 9
1 8 5 2 3 7 0 4 6 9	0 5 1 4 6 2 7 9 3 8	7 1 0 9 3 4 2 8 6 5	2 0 7 8 6 9 4 5 3 1	4 7 2 5 3 8 9 1 6 0	9 2 4 1 6 5 8 0 3 7
3 1 8 5 2 9 7 0 4 6	6 0 5 1 4 8 2 7 9 3	3 7 1 0 9 5 4 2 8 6	6 2 0 7 8 1 9 4 5 3	3 4 7 2 5 0 8 9 1 6	6 9 2 4 1 7 5 8 0 3
2 3 1 8 5 6 9 7 0 4	4 6 0 5 1 3 8 2 7 9	9 3 7 1 0 6 5 4 2 8	8 6 2 0 7 3 1 9 4 5	5 3 4 7 2 6 0 8 9 1	1 6 9 2 4 3 7 5 8 0
5 2 3 1 8 4 6 9 7 0	1 4 6 0 5 9 3 8 2 7	0 9 3 7 1 8 6 5 4 2	7 8 6 2 0 5 3 1 9 4	2 5 3 4 7 1 6 0 8 9	4 1 6 9 2 0 3 7 5 8
8 5 2 3 1 0 4 6 9 7	5 1 4 6 0 7 9 3 8 2	1 0 9 3 7 2 8 6 5 4	0 7 8 6 2 4 5 3 1 9	7 2 5 3 4 9 1 6 0 8	2 4 1 6 9 8 0 3 7 5

4	3	2	1	0 = rechts	
1 2 6 7 5 8 3 0 9 4	0 4 3 2 1 5 6 7 8 9	7 9 6 4 0 1 3 2 5 8	2 8 3 9 7 0 6 4 1 5	0 4 3 2 1 5 6 7 8 9	A 0
5 1 2 6 7 4 8 3 0 9	1 0 4 3 2 9 5 6 7 8	0 7 9 6 4 8 1 3 2 5	7 2 8 3 9 5 0 6 4 1		D 1
7 5 1 2 6 9 4 8 3 0	2 1 0 4 3 8 9 5 6 7	4 0 7 9 6 5 8 1 3 2	9 7 2 8 3 1 5 0 6 4		G 2
6 7 5 1 2 0 9 4 8 3	3 2 1 0 4 7 8 9 5 6	6 4 0 7 9 2 5 8 1 3	3 9 7 2 8 4 1 5 0 6		K 3
2 6 7 5 1 3 0 9 4 8	4 3 2 1 0 6 7 8 9 5	9 6 4 0 7 3 2 5 8 1	8 3 9 7 2 6 4 1 5 0		L 4
8 4 9 0 3 1 5 7 6 2	5 9 8 7 6 0 1 2 3 4	1 8 5 2 3 7 0 4 6 9	0 5 1 4 6 2 7 9 3 8		N 5
3 8 4 9 0 2 1 5 7 6	6 5 9 8 7 4 0 1 2 3	3 1 8 5 2 9 7 0 4 6	6 0 5 1 4 8 2 7 9 3		S 6
0 3 8 4 9 6 2 1 5 7	7 6 5 9 8 3 4 0 1 2	2 3 1 8 5 6 9 7 0 4	4 6 0 5 1 3 8 2 7 9		U 7
9 0 3 8 4 7 6 2 1 5	8 7 6 5 9 2 3 4 0 1	5 2 3 1 8 4 6 9 7 0	1 4 6 0 5 9 3 8 2 7		Y 8
4 9 0 3 8 5 7 6 2 1	9 8 7 6 5 1 2 3 4 0	8 5 2 3 1 0 4 6 9 7	5 1 4 6 0 7 9 3 8 2		Z 9

