

Introduction

Before the advent of Computer Aided Design, construction offices relied on drawing boards, pencils, rulers, slides rules and an occasional calculating machine to design buildings and analyze their structures. For novel construction types or high-stakes projects a *structural model* could be built to prove the validity of the structure.

Structural modeling methods can be distinguished into two types: direct methods and indirect methods[1,2]. Direct methods use models that are flexurally similar to the prototype.¹ Forces, moments and displacements due to applied forces and moments are measured in the model and scaled back to the prototype. The models should be built to as large a scale as possible with materials that, after scaling, mimic the properties of materials in the prototype. This requires a laboratory with balances, cathetometers, dynamometers, strain gauges etc.

Indirect methods deal with models that are much smaller and made of simple materials such as cardboard, celluloid, perspex or thin metal strips. The models are usually built to the same scale as the drawing. In indirect methods, no known forces are applied and no forces are measured.

¹ “Prototype” is used here for a real-size structure, which is still imaginary at this stage of the design process but has all characteristics of a “real-world” structure. Hopefully, one of the prototypes will finally be built.

Instead, only deformations are imposed and measured. Indirect methods can be regarded as a calculation aid: they are part of a computing process.

Background

We consider structures that are made from members that are either rigidly connected or pin-jointed to each other. The structures themselves are rigidly connected (“encastred”), pinned or resting on the ground or another fixed support. The stress-strain relationship for the members is assumed to be linear (Hooke’s Law).

In structural analysis forces are traditionally split into a horizontal part **H** and vertical part **V**, which are treated separately. Structural analysis also deals with moments **M**. A distinction is made between *external* forces, such as loads and reaction forces from supports, and *internal* forces (axial forces, shear forces, bending moments inside the structure). The external forces can cause internal forces.

Structural analysis looks for structures that are in equilibrium: the sums of the forces and the sums of the moments (to any point) should be zero. This analysis is complicated for structures that are *statically indeterminate*. To tell if a two-dimensional beam structure is *statically indeterminate*, one should calculate

$$n = a + 3 \cdot (p - k) - r \tag{eq.1}$$

in which

- a = number of reactive forces (incl. moments) at the supports (see Figure 1)
- k = number of joints (incl. those of the supports)
- p = number of beams between joints
- r = number of additional conditions (excl. supports) = $m - 1$, in which
- m = number of hinged beams

If $n > 0$, the structure is *statically indeterminate*, if $n < 0$, the structure can move around, which is usually not to be desired, and if $n = 0$, the structure is *statically determinate*. Statically indeterminate structures are not “bad”, they are just difficult to analyze.

The analysis can be performed graphically by drawing *influence lines*. An influence line shows the effect of a unit point load on a certain force or moment at a certain point if the unit load is moved across the structure. The coordinates of the influence line give the position of the unit load and the size of the effect. It is assumed that if the size of the load changes, the effect changes proportionally.

Influence lines are derived using the Müller-Breslau principle[3]:

Figure 2a shows a statically undetermined structure: $a = 1+2+1$, $k = 3$, $p = 2$, $r = 0$, so $n = 1$. A unit load 1 is placed at a random point P . The influence line for the vertical reaction force V_A at point A is obtained by the following steps. Remove the roller bearing at A and replace it by an (unknown) force V_A which causes the deflection at point A to be zero (Fig. 2b). Place a unit load 1 at a random point P and keep V_A zero (Figure 2c). This will deflect A by δ_{AP} and P by δ_{PP} . Remove the force at P and apply a unit reaction force at A (Figure 2d, note the change in direction!) Now A will be deflected by δ_{AA} and P by δ_{PA} . For the deflec-

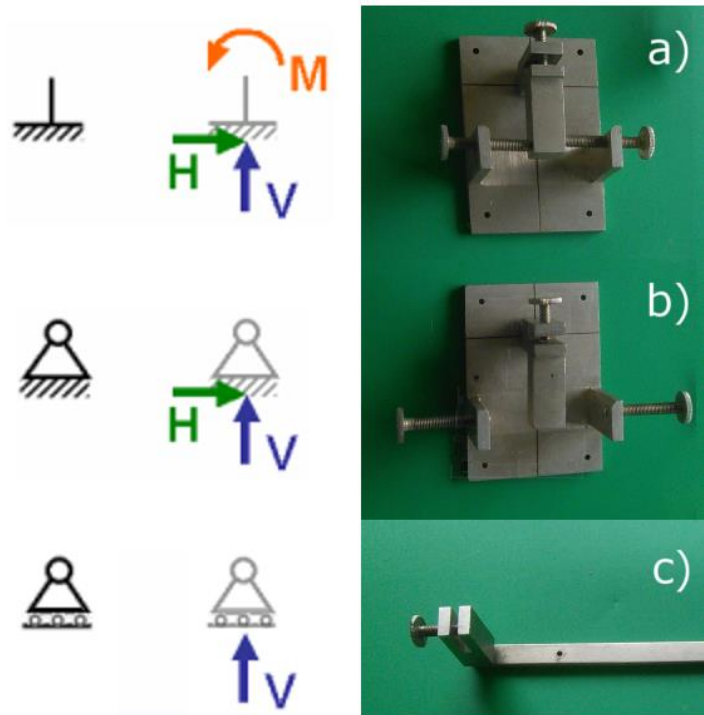


Figure 1: Encastred (a), pinned (b), and rolling (c) supports, with NuPuBest equivalents

tion Δ of A to be zero if a load of 1 is applied at point P (as in Figure 2b), the reaction force V_A should be $-\delta_{AP}/\delta_{AA}$. Now the Maxwell-Betti reciprocity theorem is used, which states that, for linear-elastic structures, the deflection at point P due to a unit force at point A equals the deflection at point A due to a unit force at point P. In other words: $\delta_{AP} = \delta_{PA}$ so $V_A = -\delta_{PA}/\delta_{AA}$. This means that the influence line for the reaction force at A (caused by a unit load at any point P) corresponds to the deformed structure that is obtained by displacing point A from its equilibrium position (Figure 2e). No explicit measurement of forces is needed!

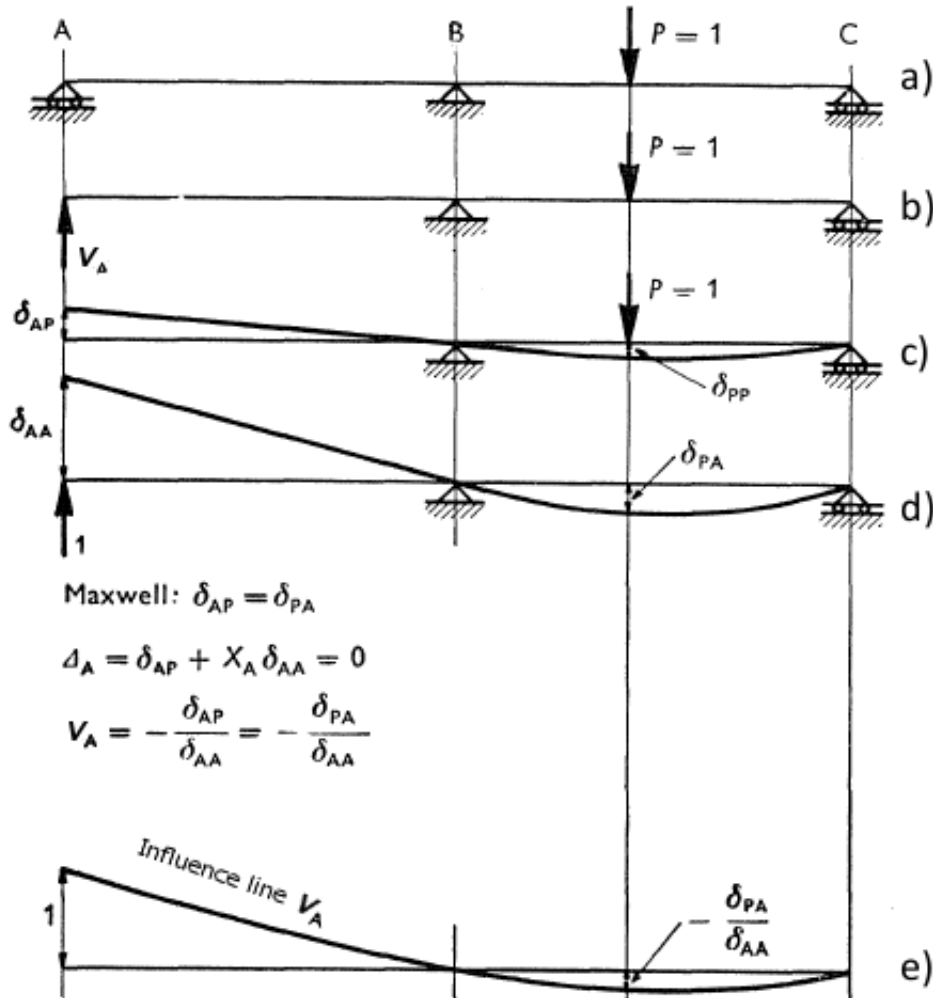


Figure 2: Derivation of the Influence line for V_A

Early history

In the second half of the 19th century several graphical and computational methods were developed for the complete analysis of statically indeterminate structures.[4] These methods were tedious and often relied on trial-and-error. Later, in 1930, H. Cross published the iterative “distributed moment” method[5], which improved the chances of getting an analysis done in a reasonable time.[6]

Meanwhile, in 1916, G.E. Beggs, of Princeton University, developed a method using two-dimensional cardboard or celluloid models with metal “deformator gauges” and measuring microscopes[7,8, 30] (Figure 3). The deformator gauges are clamps in which calibrated gauge plugs (with tolerances of 0.0002 inch!) can be inserted to cause displacement, shear or rotation. The model is placed horizontally, with some gauges screwed to the supporting table and parts of the model riding on tiny balls to reduce friction. The system is sensitive to temperature changes and vibrations, as illustrated by the position of the operator’s hands in Figure 4. It is also said to cause considerable eye strain.[2]

W.J. Eney, of Lehigh University, developed a cheaper variant of the Beggs’ deformeter in 1939.[9]

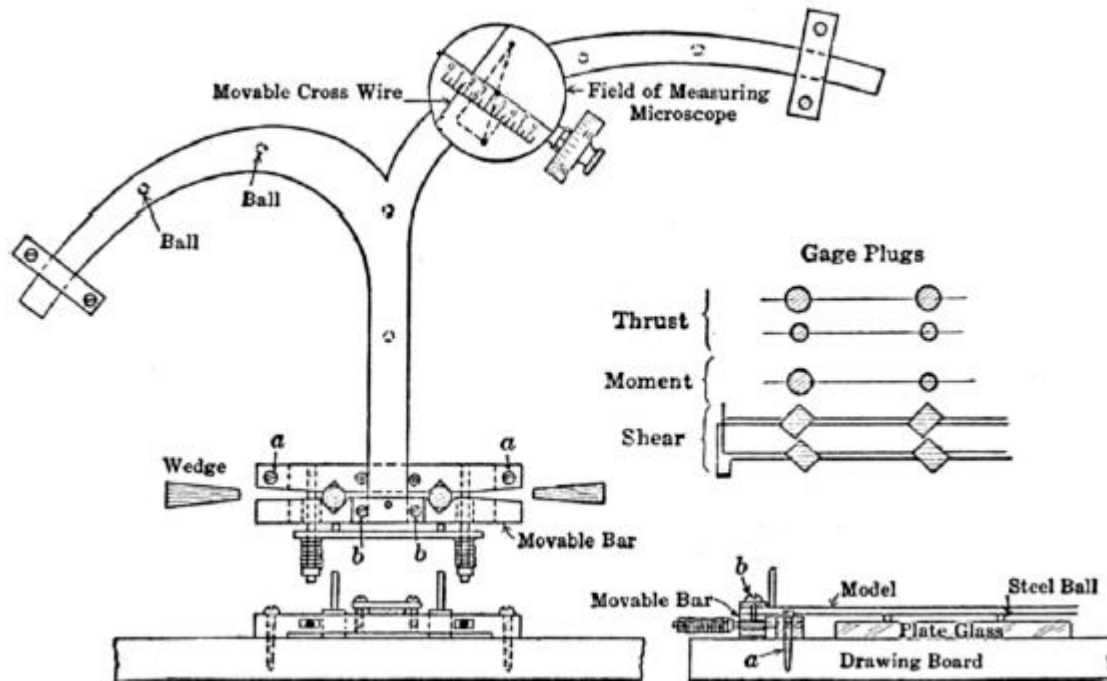


Figure 3: Sketch of Beggs' deformer apparatus [10]

Continostat

The disadvantages of Beggs' method led Otto Gottschalk, from Buenos Aires, to propose another modeling method: one using very large deformations on a vertically mounted model.[11] The model uses steel splines as building material, and therefore does not resemble the prototype as well as Beggs' shaped celluloid models. A heavy horizontal rule is used as a base, on which clamps are placed that hold struts that support the model. The German patent 380,528[12] describes the struts as having a rack and pinion to deform the model. This seems to be omitted in the actual device.[13] Small pulleys and a piece of rope with two weights can be used to impose forces, and thereby deformations, on the model.[12,14] Gottschalk made two versions of his device[15]: the Continostat and the Continostat-A or Mechanostat. The latter contained a larger variety of linking elements and could also be used for modeling ships, airplanes, engine parts etc.[16]

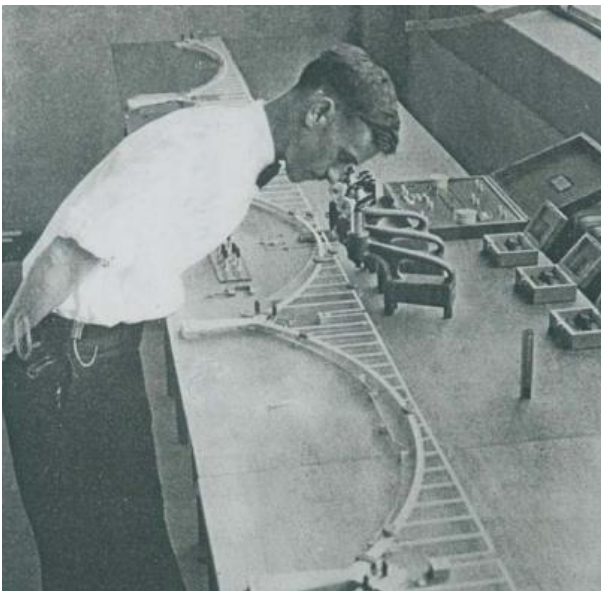


Figure 4: Using a Beggs deformer [27]

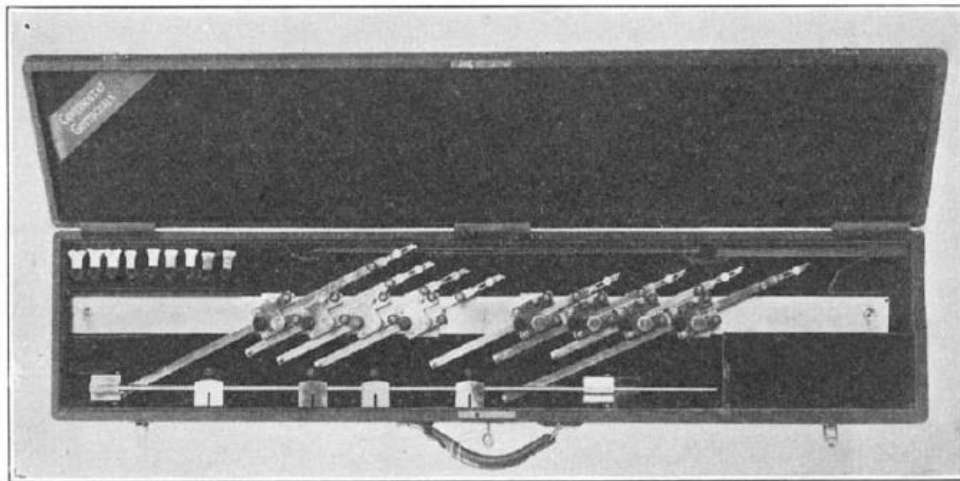


Figure 5: Continostat [15]

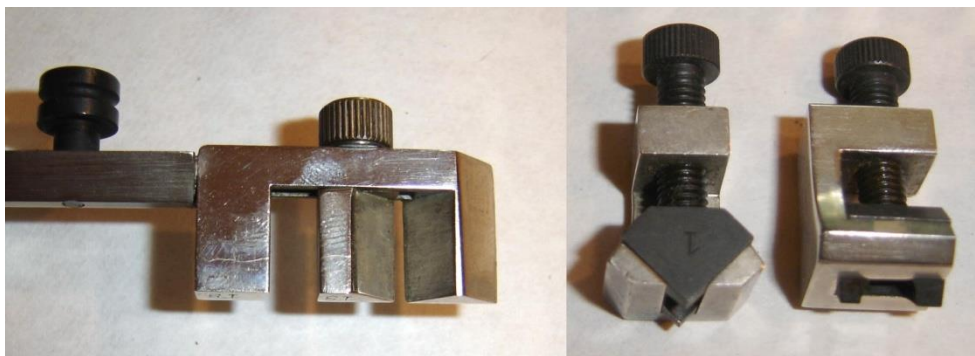


Figure 6: Continostat clamps

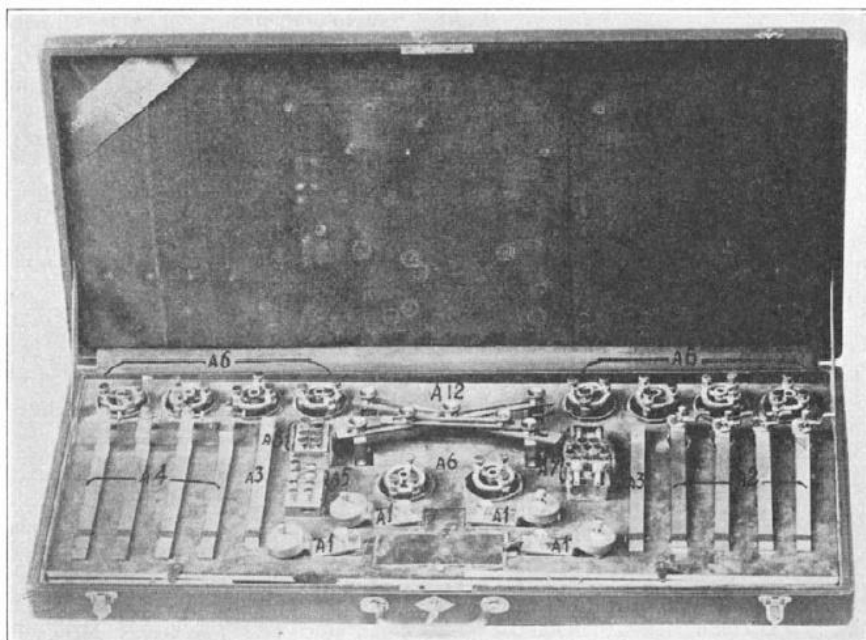


Figure 7: Continostat-A [15]



Figure 8: NuPuBest: first version (left[28]) and third version (right)

NuPuBest

Shortly after the invention of the Continostat[17], Christian Rieckhof proposed the NuPuBest (Figure 8). The NuPuBest also uses simple metal splines. Links between the splines are made using cross-cut metal cubes with screws or small cups on which clamps can be fixed (Figure 9). Small balls are put under the links to reduce friction.



Figure 9: Basic NuPuBest links

The model is built in a horizontal plane on a drawing board using drawing tacks to pin the model supports to the board. The supports are shown in Figure 1. The rolling support is a long strip, with one end pinned to the board and the other, movable, end carrying a swiveling clamp for the spline.

The name “NuPuBest” stands for “Null-PunktBestimmung” (Zero-point finder) which indicates a special feature of the device. While Beggs and Gottschalk fo-

cused on drawing influence lines, Rieckhof equipped the NuPuBest with a curvimeter (Figure 10c) to find the points of zero curvature in the deformed structure. At points of zero curvature moments are zero. A statically indeterminate structure might be cut up at these points into a set of (hinged) determinate structures (remember the r in equation 1). This simplifies the numerical analysis of the structure.

The curvimeters are standard devices as used in the optical industry. They have scales and are sometimes marked with a refractive index of glass, but for the NuPuBest only the “zero” mark is significant. The position along the spline of the zero deflection is marked on the drawing. Because there is some distance between the spline and the drawing, Rieckhof recommends applying one of the unused construction elements as a plumb line. Instead of marking the points, they can also be measured by the tangent pointer (“Tangentenzeiger”, Figure 10b) which can be clamped to a reference point on a spline.

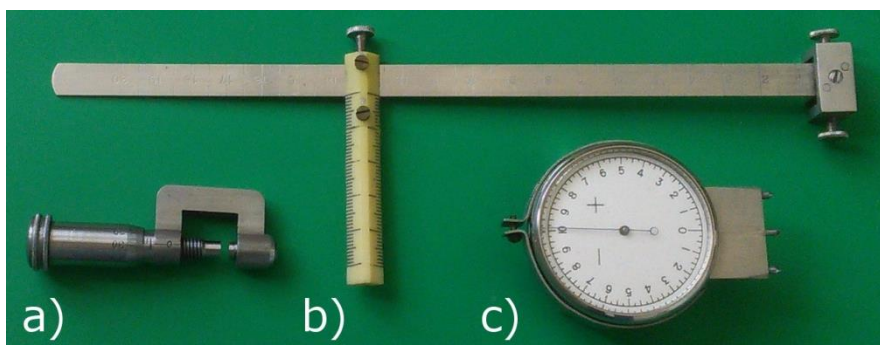


Figure 10: NuPuBest measuring devices

EFluBest

Influence lines could also be drawn with the model. Rieckhof called this the “EFluBest” method (Ein-FlusslinienBestimmung).

Both methods use large displacements, like the Continostat. However, the structural analysis is based on linear

elasticity theory, leaving out second and higher order terms. The model displacements are so large that second order effects cannot be neglected. The solution is to deform the structures twice, in opposite directions, and use the absolute sum of the applied and measured displacements, which cancels the second order terms². The shape of deformed spline is usually drawn onto the drawing board. Rieckhof gave detailed instructions on how to apply the deformations, and even patented them.[29] In some cases a temporary fixation of some of the links is required, for which long hat pins are used.

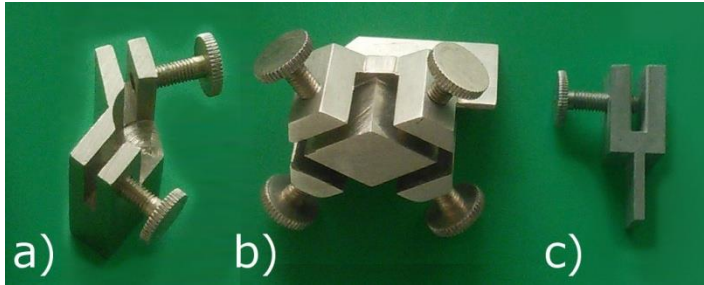


Figure 11: Special NuPuBest links

The influence line for an internal moment of a beam is determined by replacing the corresponding spline by two similar shorter splines and connecting them by a special bent link (Figure 11a) which has an angle of 123° (the complement to 1 rad). The influence line for a moment at a joint is obtained by less radical change of the model: a special rhomboid link (Figure 11b) is placed in a basic link instead

of one of the splines. This spline is put in one of the slots of the rhomboid, and thereby rotated but also lifted. The other end of the spline also has to be lifted, by putting the bottom of a simple clamp (Figure 11c) into its basic link, and locking the spline in this clamp. The disadvantage of this procedure is that, because the spline is lifted further from the drawing board, transferring its shape to the drawing board is more difficult.

The splines are not provided with the NuPuBest: the user should acquire them separately. These splines should be 0.5 to 1.0 mm thick, 10 mm wide strips of spring steel (“Federbandstahl”). If the prototype consists of beams with different cross sections, giving different moments of inertia, the model should be built from strips of various thicknesses. For this purpose a micrometer gauge is included in the NuPuBest set (Figure 10a).

Example

A prototype can be analyzed using both methods (NuPuBest and EFluBest) with just one model.

Take for example a horizontal beam resting on 4 pinned supports A , B , C and D (Figure 12). This structure is statically indeterminate: $n = 2 + 2 + 2 + 2 - 3 \cdot (3 - 4) + 0 = 5$.

Instead of the vice-like support of Figure 1b, we use the strip-support of Figure 1c, but now each strip (b in Figure 12) is fixed to the drawing board with two pins c . Each strip carries a clamp d that can freely rotate around a vertical axis.

Let’s assume we need to know the vertical reaction force at support B when a unit downward load is put at point P , 2 meters to the right from B . For the NuPuBest method this means pulling down the beam at P using a clamp on an additional strip g , after loosening all screws in the clamps d at A , B , C and D . During the pull-down, the spline representing the beam can shift and deform, and the clamps d may rotate. The pull-down distance does not really matter: about one-tenth of the beam length is fine. Finally the structure is fixed by pinning down the strip g and fastening the screws of all d ’s. Now one can start searching for the points of zero curvature using the curvimeter. There are two zero-points a , near B , and a' , near C . They are marked on the drawing board.

Next the screws of the clamps d are loosened again, and the strip g is shifted up twice the distance it was shifted down, giving a mirror deformation in which also two zero-points a and a' can be found. The “positive” and “negative” zero-points are interpolated pairwise, giving the zero-points of the undeformed structure. With this information a classical analysis for three *determinate* structures³ can be performed.

² Strictly speaking, for force-influence lines one should project the displacement onto the direction of the force.

³ The 3 structures are only determinate for vertical forces, which is what we happen to be interested in.

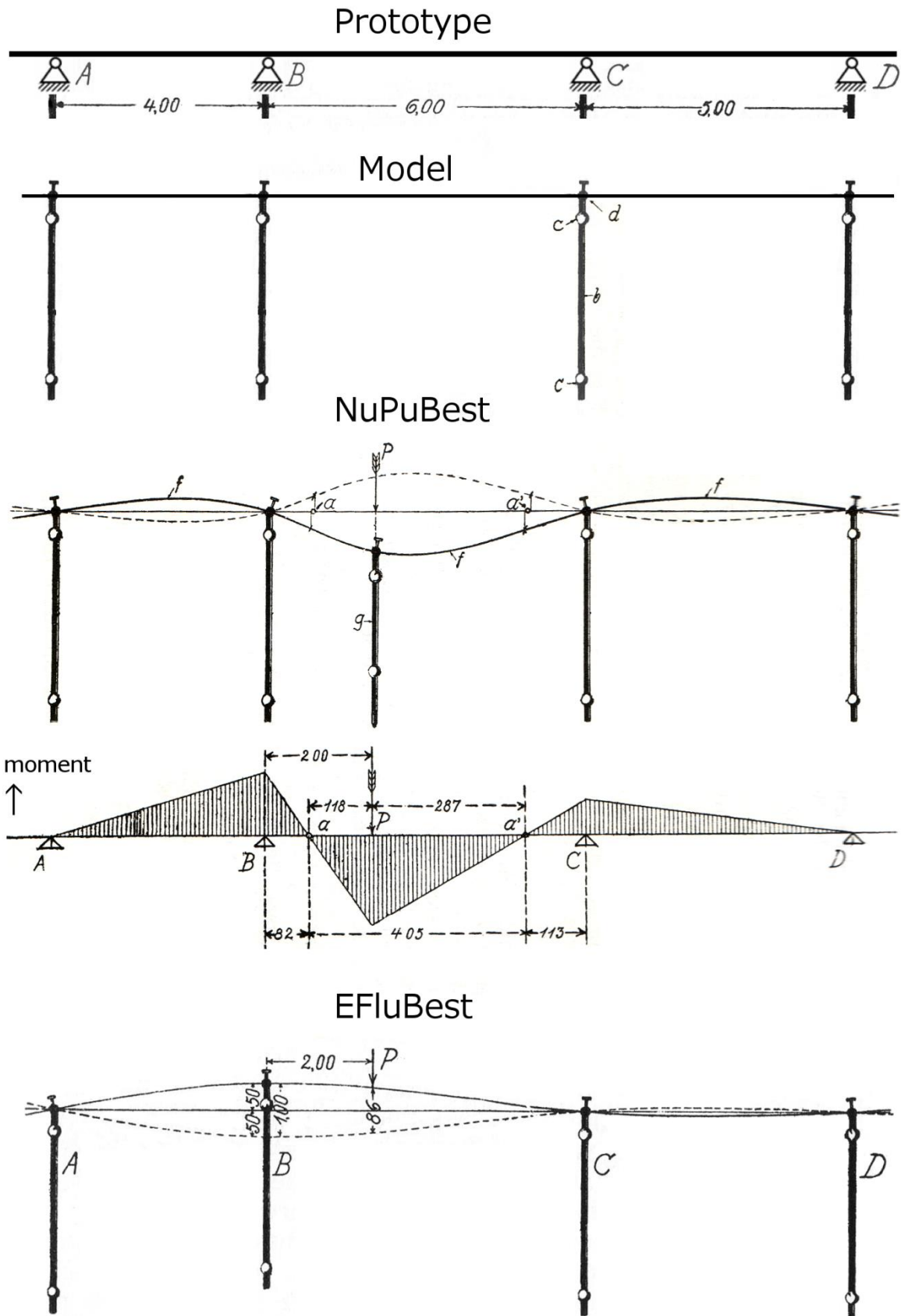


Figure 12: Example

The structure between a and a' is a simple beam resting on two pinned supports (which have non-zero reaction forces and zero moment). The load is distributed over the two supports: the vertical reaction force V_a at support a equals $|Pa'|/|aa'|$ and the reaction force at a' is $|aP|/|aa'|$.

We now turn to the second structure, the one containing A , B and a . The moment M_B at B becomes $V_a \cdot |Ba| = |Ba| \cdot |Pa'|/|aa'|$. At B there are two contributions to the reaction force: one from the right, equal to V_a , and one from the left, equal to $M_B/|AB|$. So the vertical reaction force V_B at B for a unit force at P is $|Pa'|/|aa'| + |Ba| \cdot |Pa'|/|aa'|/|AB|$.

In the current prototype that would be $V_B = 0.854$. To check if all signs are OK a moment plot is drawn, such as the fourth drawing in Figure 12.

For the EFluBest method we deform the structure by shifting support B up and down (we are still interested in the vertical reaction force at B). The ratio between the vertical components of the deformation of the structure at P and B gives the reaction force at B for a unit force at P . If the total deformation at B equals 1, we read directly from the model: $V_B = 0.86$, which is quite close to the calculated value. If we need V_B for a unit load at another point P' , we can also read that value directly from the deformation at point P' . It is obvious the EFluBest method is much faster than the NuPuBest and gives more results in one go.

Versions

There are three known versions of the NuPuBest. The first version consisted only of the construction set[28]. The position of the zero point should be derived graphically from the deformed model.

In the next two versions different curvimeters are added. The old curvimeter is mounted in a three-legged ring while the newer one is being balanced on the spline (Figure 13). The newer NuPuBest has a magnetic strip to put the friction-reducing balls into position, while the older ones provide a simple cardboard fork for that purpose (Figure 14).

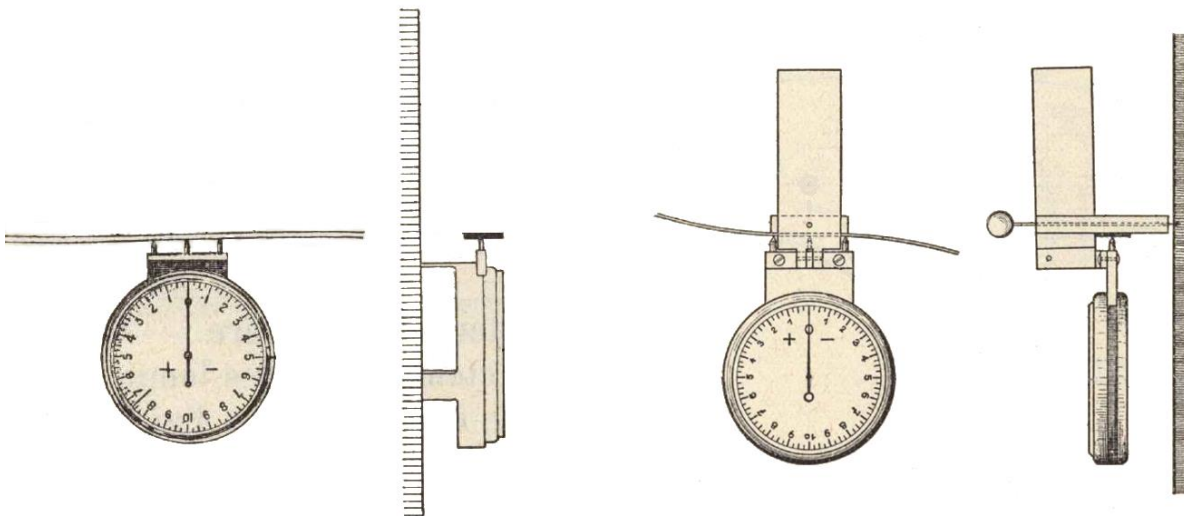


Figure 13: Two versions of the NuPuBest curvimeter: old (left) and new (right)

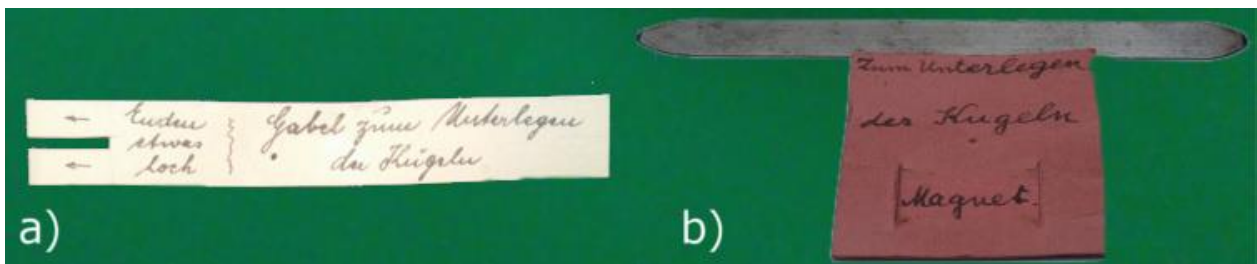


Figure 14: Old (a) and new(b) devices for putting NuPuBest ball bearings into place

Commercial history

The NuPuBest was originally sold by the Aktiengesellschaft für Baubedarf, Ludwigsstraße 15, Darmstadt[18]. Around 1927, this company, of which Rieckhof was “Direktor”[17], ceased to exist and the NuPuBest was sold privately by Rieckhof, then living at Moosbergstraße 97, Darmstadt. Rieckhof wrote a series of textbooks on the NuPuBest and EFlubest that were partly published privately[18] and partly by technical publishers (VDI, Editions du Constructeur de Ciment Armé[19]). In the early 1960’s the NuPuBest was sold by E. Gerdenitsch, Roquetteweg 45, Darmstadt, who also published the manual “Experimentelle Statik” privately. It is unknown when Gerdenitsch took over business. We only know that a 1958 letter to “Chr. Rieckhof (or Successor), Moosbergstraße 97, Darmstadt” ended up in the archive of Gerdenitsch. Beggs’ and Gottschalks devices were made commercially, but details are lacking.

Practical use

One might wonder if these modeling devices were actually applied in practical calculations, or were merely used in an educational context. Almost all contemporary accounts[20] mention the value of the models as a teaching aid. In fact, the Beggs deformer is still being used in engineering courses[21] and is still being manufactured[22]. Continostats show up in various university collections and old course descriptions[23]. E. Gerdenitsch, the NuPuBest dealer in the late 1950’s, only gives testimonials from university professors in his advertising pamphlets. This does not rule out commercial construction offices as customers, maybe it is just good German marketing. In fact, the second hand NuPuBest the author acquired originated from a pre-WWII Yugoslavian construction office.

The practical use of the NuPuBest is also indicated by the acceptance of the combined NuPuBest-EFluBest method by the Prussian Building Authority (Preussische Baupolizei)[24]. However, while it is easy to find references to Beggs’ deformer in actual use, only four references have been found to construction projects that used the NuPuBest[25, 26].

The Delft Connection

In 1956, D. Reinders of Delft University of Technology improved Rieckhof’s modeling method by simplifying the clamps, in order to minimize the distance between the splines and the drawing board[6]. He also improved modeling with flat-lying perspex strips (figure 15) with or without “internal joints” that were simplified versions of Beggs’ deformer gauges, but now for large displacements.

Reinders pointed out that small sections of additional stiffness introduced by the clamps could result in large errors. The Continostat clamps are either knife edges, causing no additional stiffness, or flat clamps, causing maximum stiffness (Figure 6). The NuPuBest clamps are in between: a screw pressing the spline against a flat plane. But because the NuPuBest and EFluBest methods both require deformations in two opposite directions, additional stiffness cannot be avoided. This might be another reason the more “exact”, but also more expensive, Beggs deformer overshadowed the NuPuBest-EFluBest and the Continostat.

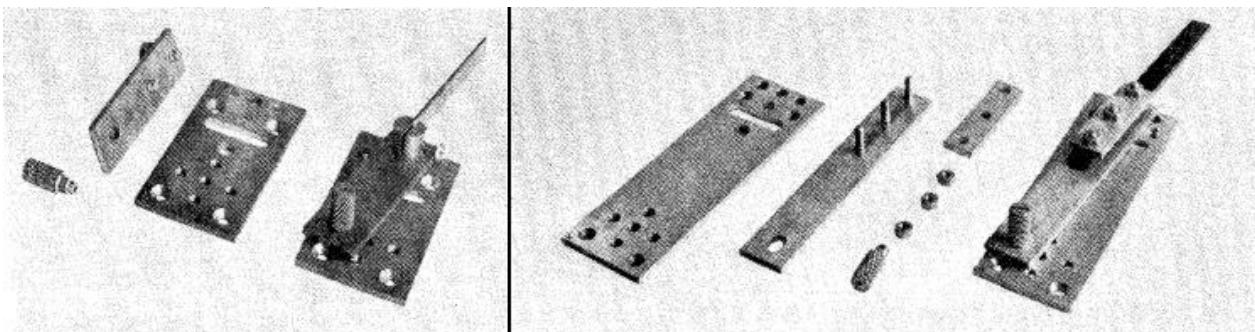


Figure 15: Reinders' clamps for metal (left) and perspex (right) models

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