DR HARO'S NEW METHOD OF GRAPHICAL NOTATION OF LOGARITHMS

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A few years ago, while exploring the national French digital library (http://gallica.bnf.fr/), I came across the proceedings of a conference[i] held in Toulouse the 24th of September 1887 by a certain Dr Haro, medical officer in the French army. In this conference Dr Haro submitted to the *Association Française pour l'Avancement des Sciences*[ii] a logarithmic circle of his invention and described how to use it.
2014 being the quadricentenary anniversary of the invention of logarithms by John Napier, I found it interesting to shortly describe this peculiar circle which is in fact nothing else than a portable analog table of logarithms, allowing to read the mantissa of a number up to 4 decimals.

Front face
The front part of the instrument consists of a disk, whose limb is divided in one thousand equal parts and on which ten concentric circles are drawn. The reading of the mantissa is done thanks to an alidad designed with an extending piece bearing a vernier.
The principle of use is quite simple:

1. Set the edge of the alidad on the number printed on one of the concentric circles.
2. Read the first decimal of the mantissa by spotting the order number of the circle on which the number is printed.
3. Read the second and third decimals on the limb.
4. Finally read the fourth decimal thanks to the vernier.

Obviously, as mentioned by the inventor:

> *by increasing the number of circles, it would equally be possible to obtain the fifth or even the sixth decimal if the limb would be big enough to allow the reading of ten-thousandths of the circumference*.

Back face
The limb is also divided in equal parts but the number of concentric circles is higher, 33 in total. 22 of them are used for reading the logs of tangents and the rest are used for reading the logs of sines.
The circles corresponding to the tangents are drawn nearest to the centre and only from 5° to 85°. As the logarithms of tangents show higher differences than the logarithms of sines, especially near 90°, the circles corresponding to the sines are drawn nearest to the limb.
The reading of the mantissa is similarly done thanks to an alidad with a vernier.

Construction
Dr Haro described quite precisely how he constructed the prototype exhibited during the conference. For that purpose, he used the mantissas found in *Callet's* table of logarithms.

> *After having divided in hundred equal parts a circumference whose diameter is 0^m15, and having named every five of these divisions 50, 100, 150,... 1,000, I have drawn ten concentric circles bearing the digits 0, 1, 2, 3,... 9, beginning from the centre.*
> *Then, with the help of a cardboard alidad, which had one of its edges coinciding exactly with the radius and was secured at the centre of the disk by a pin, I have inscribed the logarithms of the numbers on the ten circles by using a vernier fitted on the alidad. The vernier allowed me to read the tenths of the divisions of the limb. I proceeded as follows.*
>
> - *The log of 1 being 0,000, I set my alidad on the 0 division of the limb and draw a graduation mark on the circle indicated 0, then I write down a bold digit 1 opposite this mark.*
> - *The log of 2 being 0,3010, I set the alidad on the 10 division of the limb and draw a graduation mark on the circle indicated 3, then I write down a bold digit 2 in front of this mark.*

- *The log of 3 being 0,4771, I set the alidad on the 771 division of the limb and draw a graduation mark on the circle indicated 4, then I write down a bold digit 3 in front of this mark.*
- *Etc. up to 9.*

….To somehow strengthen my drawing, I glued it on a tin plate provided with a salient edge that is meant to protect the divisions of the limb against any deterioration. I also replaced the cardboard alidad by an alidad made of horn fitted with a vernier and an index drawn exactly in the direction of the radius. This transparent alidad makes the reading remarkably easier and bears the numerical order of the circles; it is secured on the centre of the disk by means of a steel pivot fitted with a button that makes it possible to easily move the alidad in both directions, as the case may be.

My own prototype

Following Dr Haro's instructions I produced Figures 1 to 12 with the aid of a CAD program. They show the operating mode to print (or engrave) the front face of the circle. The illustrations are scaled to about 4/10th of the real size.

The alidad is coloured in only for the sake of readability of the present article.

- The first step is to divide the limb in equal parts. Figure 1 shows the division in 100 equal parts but ideally it should be divided in thousandths.
- Figure 2 shows the ten concentric circles intended to bear the numbers. For the clarity I have left the digits showing the order number on each figure but in fact these digits only need to be printed on the rotating alidad.

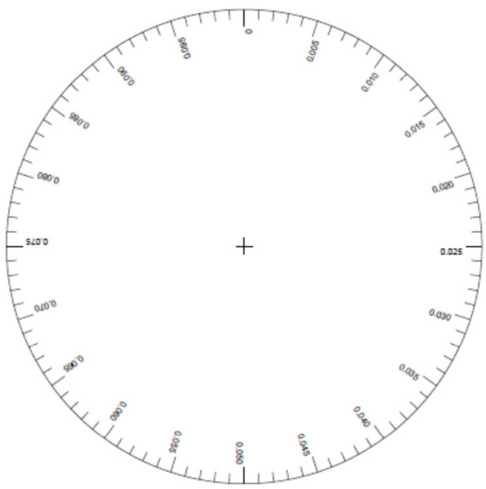


Fig 1. Division of the limb in equal parts

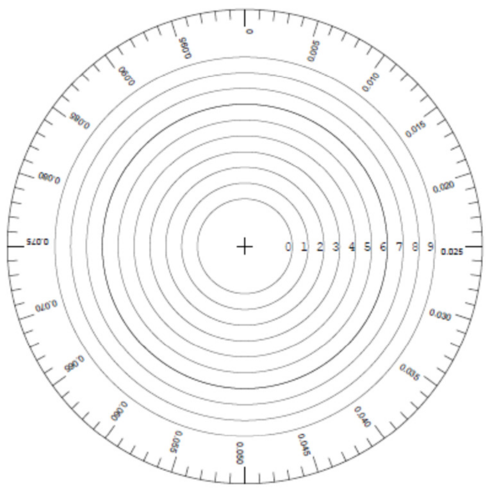


Fig 2. Drawing 10 inner circles

- Figure 3 shows how to position the number 1 on its corresponding circle: log of 1 being equal to zero it suffices to set the alidad at the zero graduation of the limb and to print the graduation mark for the number 1 on the circle named zero.
- Figure 4 shows how to position the number 2: log of 2 being equal to 0.301, the alidad is set at the 0.001 graduation of the limb (second and third decimals of the mantissa) and the graduation mark for the number 2 is printed on the circle named 3 (the first decimal of the mantissa being 3).

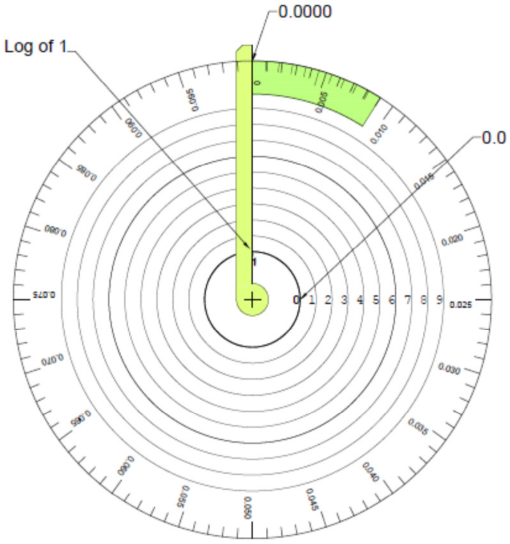


Fig 3. Logarithm of 1 = 0.0000

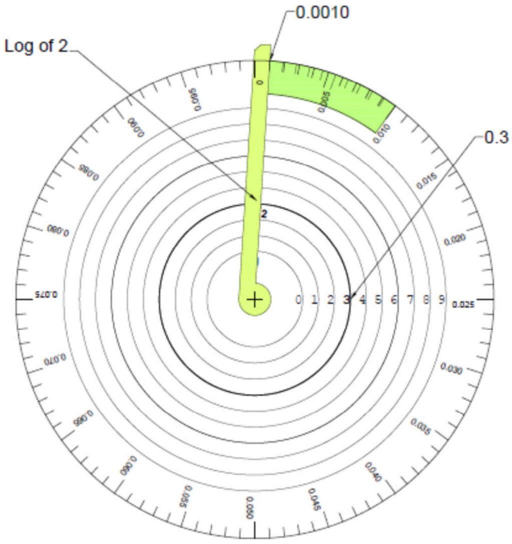


Fig 4. Logarithm of 2 = 0.3010

- Figures 5 to 11 show similarly how to position the graduation marks for the numbers 3 to 9 on their corresponding circles.

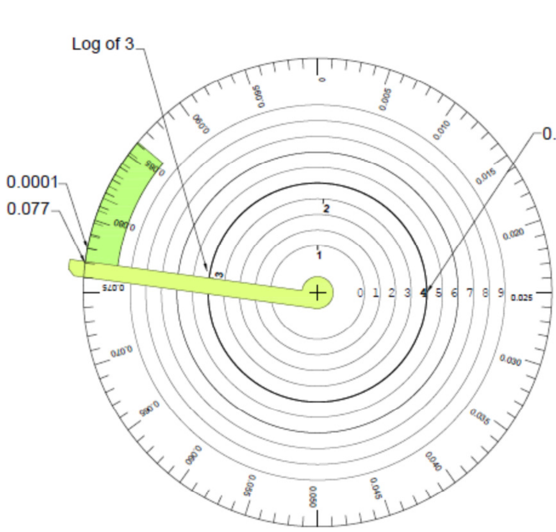


Fig 5. Logarithm of 3 = 0.4771

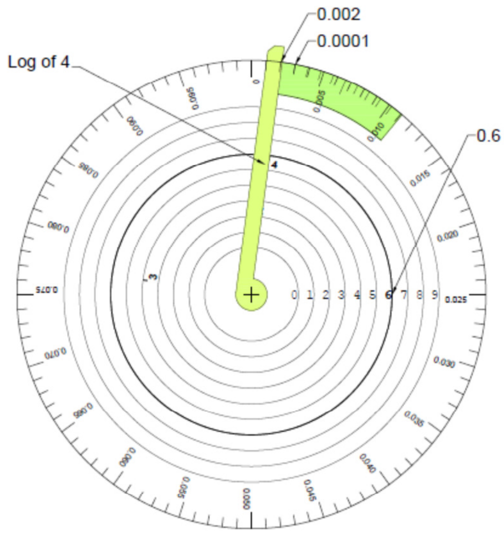


Fig 6. Logarithm of 4 = 0.6021

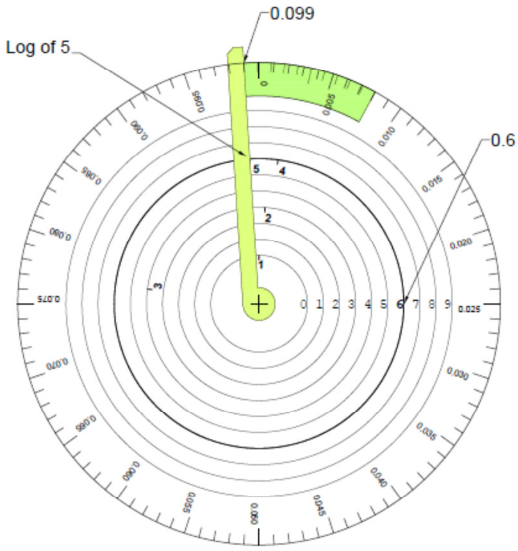


Fig 7. Logarithm of 5 = 0.6990

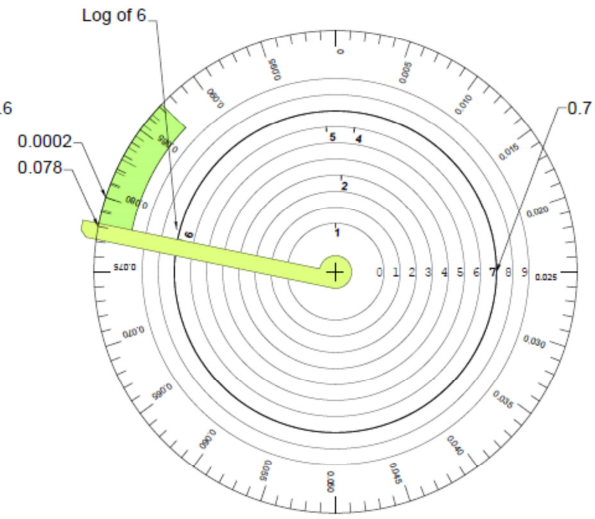


Fig 8. Logarithm of 6 = 0.7782

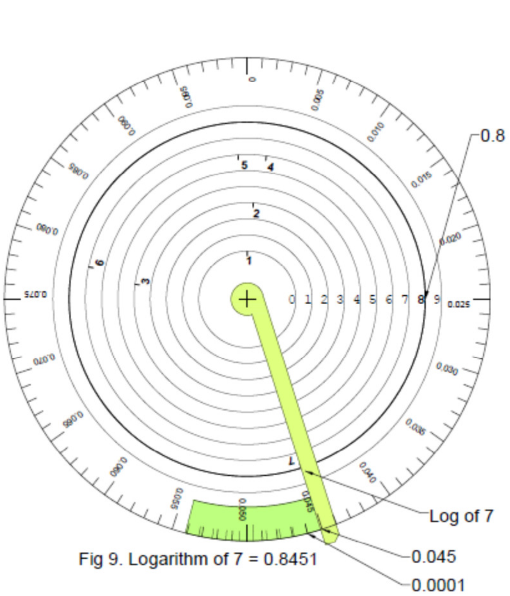


Fig 9. Logarithm of 7 = 0.8451

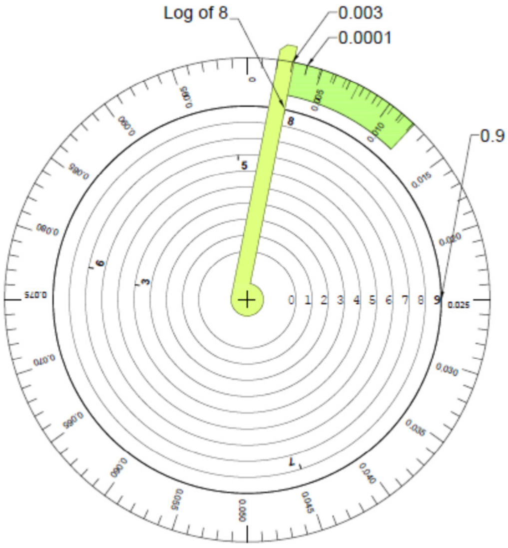


Fig 10. Logarithm of 8 = 0.9031

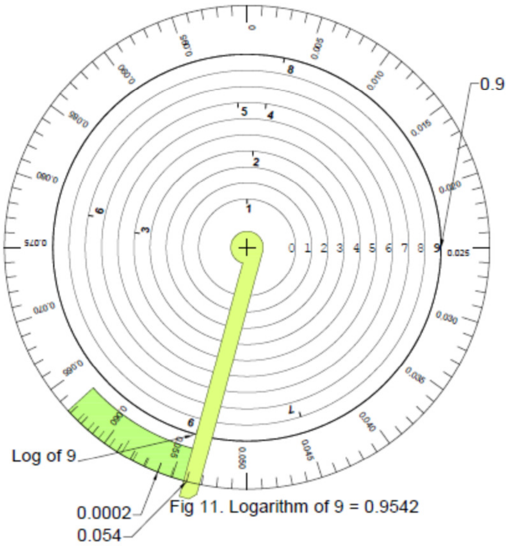


Fig 11. Logarithm of 9 = 0.9542

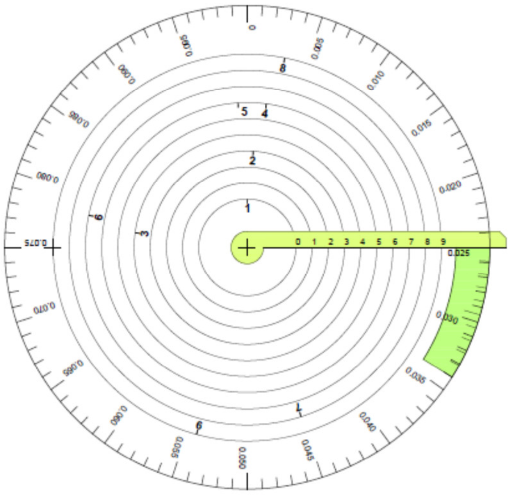


Fig 12. Alidad with vernier and indication of the circles' order number

- Figure 12 shows the instrument with the alidad bearing the order number of the circle for the ease of reading.

Prototype

The prototype shown at the conference by Dr Haro had been hand-made by himself. Unfortunately, the proceedings of the conference do not include a picture of Dr Haro's circle, so I had myself to «cobble something together» in order to understand its construction and use.
Figure 13 shows about half scale the front face of my version of *Dr Haro's circle*. It is made of printed paper glued on a cardboard and bearing an alidad made from a transparent film. I have limited myself to the construction of the front face only. The alidad is set to read the mantissa of 14 = 0.1461. If you are observant you might have noticed that the graduation marks on Figures 1 to 12 are named using a decimal notation, i.e. 0.005, 0.010, 0.015, 0.020, etc. in the contrary to Figure 13 where Dr Haro's notation is used, i.e. 0, 50, 100, 150, 200, etc. This is due to the fact that, after having made my prototype, I discovered that the decimal notation was better for the ease of reading the results.

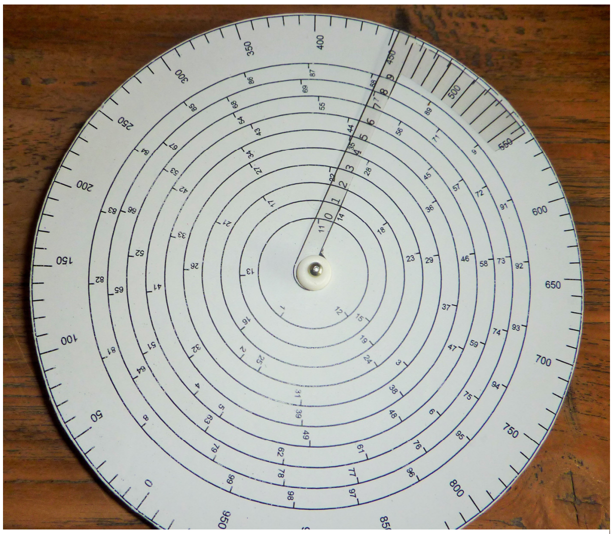


Figure 13. My own hand-made prototype

Use of the circle

According to Dr Haro, the best way to use the circle is to grip, with the left hand, the button opposite the face one wants to use —the alidad always pointing upwards— and to rotate the circle until the required number is set at the edge of the alidad (see Figure 13). One writes then on paper —as the readings proceed— the digits read from the circle, and of which the entirety forms the sought logarithm.
The results obtain with my prototype were quite accurate but I must say it is not easy, at a glance, to spot the correct number on the concentric circles, and that with a prototype valid only for reading the mantissas of 99 numbers.

Final word

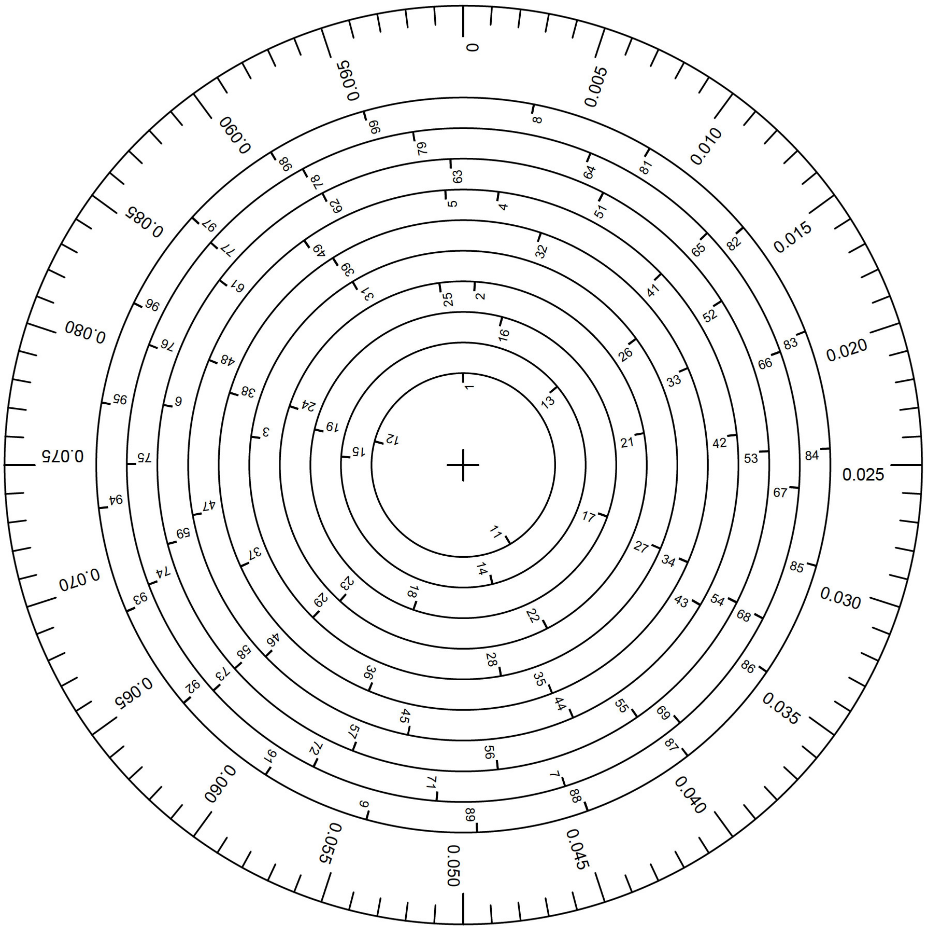
Quote from Dr Haro's conference:

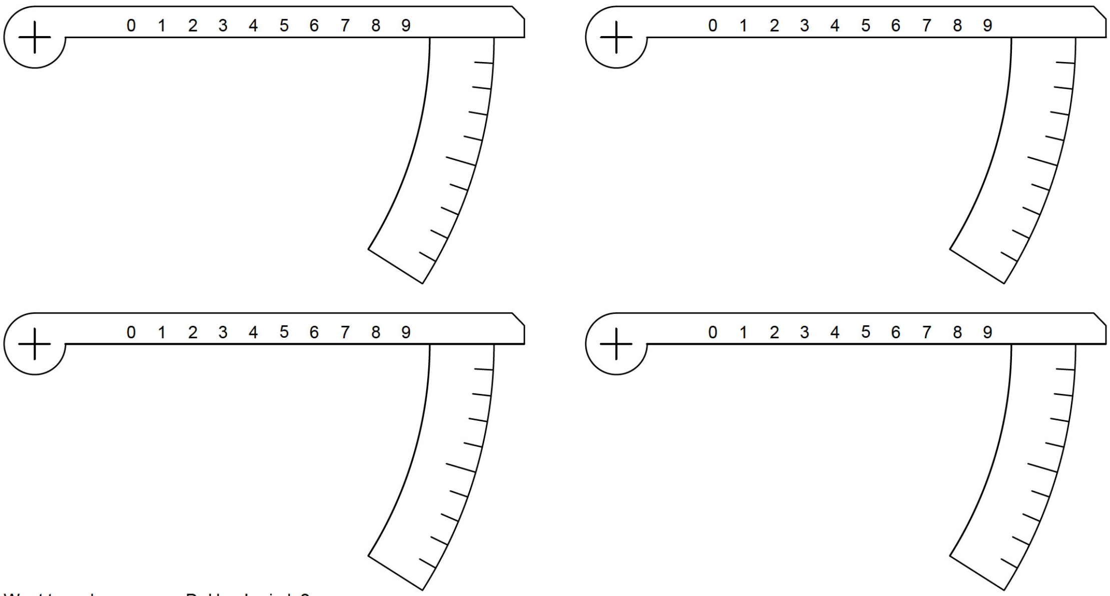
> *This circle is meant to be a precision instrument but, being poorly built by myself, it is obviously imperfect; nevertheless I hope it is good enough to show how useful it would be if it was appropriately engraved.*

I, for certain, enjoyed making my own prototype and «playing» with the circle but, though it is based on a clever idea, I am quite sure it was never commercially constructed except for some prototypes.
But, hey, we can still salute Dr Haro. After all, this guy invented something clever and there is another guy, almost 130 years later, still talking about his invention. Not anybody can make a similar claim!

i http://gallica.bnf.fr/ark:/12148/bpt6k201167p/f131.image.r=, p. 128 : *Nouvelle méthode de notation graphique des logarithmes.*

ii French Association for the Advancement of Science.





Want to make your own Dr Haro's circle?
1. Photocopy this page (on paper for the circle and on transparency film for the cursor).
2. Glue the circle part on cardboard and cut the circle along the line.
3. Cut the cursor; you should be able to get away with it, there are four of them and you only need one!
4. Use your imagination and find something existing to hold them together (I used shelf pins but you can use a boardpin or anything similar.)
5. Enjoy !