CALCULATING DISC BY BENJAMIN AYRES (circa 1750) Otto van Poelje

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Introduction

In the museum quarter of the Dutch city of Utrecht, one landmark among the old buildings is very recognizable: a modern structure called the “Glass Box”. In the early 1990s, architect Koen van Velsen rebuilt the original Laboratory of Botany of the University of Utrecht into a Science Museum, the University Museum of Utrecht (UMU). Like so many university museums in the world, the UMU preserves the instruments that were used over the past centuries in the university laboratories for research and demonstration purposes. In our own field of interest, the UMU collection of some 20 proportional dividers is beautiful, but other calculating instruments too are well represented. Most of the 100 slide rules and calculators in the UMU are 20th century, but some are pre-1900: three sets of Napier bones and some 16 slide rules and discs. One of the oldest is subject of this article, item ME32: *“Rekenschijf, Ayres B”.*

Figure 1: Front of the Ayres disc

Figure 2: Back of the Ayres disc

Description

The calculating disc is made of brass and its diameter is 15.5 cm, see figure 1. There are two rotating pointers, each with a stranded metal-wire hairline to allow an unobstructed view of scale divisions close to the hairline (as opposed to the older “knife-edge” pointers). Each of the two pointers can be clamped in position by a screw at the back. No inscriptions exist on the back side at all, see figure 2.
This disc calculator follows the principle of Oughtred's earliest calculating disc (1620s), with circular logarithmic scales on a stationary disc and two friction-coupled rotating pointers.
The two pointers can be set at an angle to each other to reflect a proportion of two values on one of the circular scales. Then the combination of the two pointers -with constant angle between them- can be moved around the scales to use this proportion in multiplications or divisions along the same or a different scale.

Scales

The circular scales on the disc (see figure 3), all of them logarithmic, are listed from outside to inside; the sequence of values is given anti-clockwise over the full circle (the abbreviations are added by the author for easier reference):

G : "Girt Line", a one-decade log line, marked 10-12-20-30- . . . -90-10
GI : "Girt Line", an inverse one-decade log line, marked 10-90- . . . -30-20-12-10
N : "Num", a two-decade log line, marked 1-2-3- . . . -9-1-2-3- . . . -9-1
T1 : "Tan", the lower tangent scale, marked 34′ 22.8″ to 45º (corresponding tangent values 0.01 to 1 on N-scale)
T2 : "Tan", the higher tangent scale , marked 45º to 89º 25′ 48″ degrees (corresponding tangent values 1 to 100 on N-scale)
S : "Sin", sine scale, marked 34′ 22.8″ to 90º (corresponding sine values 0.01 to 1 are on N-scale)
SI : "Sin", inverse sine scale, marked 90º to 34′ 22.8″ (corresponding sine values 1 to 0.01 are on N-scale - the inverse sine scale represents the cosecans function as well)

The N-scale has a radius of 6.5 cm, so its stretched length is about 40 cm. Precision of the N-scale in the high end (5 to 10) is 5 divisions per 1000, comparable to a desk slide rule with a 25 cm scale. The goniometric scales are subdivided per degree in sexagesimal units, for example one division per minute in the low range up to 5º, one division per 2 minutes up to 10º, and further decreasing precision for greater angles. Some individual division marks appear slightly misaligned, clearly due to manual dividing.

Figure 3. Scales on the Ayres disc

These scales were well-known in generic slide rules for multiplication, division, squaring, square roots and goniometric calculations. The use of inverse scales was introduced only much later on regular slide rules, for example in improved Rietz slide rules, early 20th century. The addition of such inverse scales allows easier operation of a combined multiplication and division.

Girt Line for Timber Measurements

The name "Girt Line" (for scale G and GI) is well-known from the scales of the "Carpenter Rule", as described by Henri Coggeshall (1623-1690), and pictured in his posthumous pamphlet [2].
The purpose of the Girt Line was to calculate the volume V of a felled tree by multiplying length L of the trunk with the area A of its circular cross-section. To determine this area, a string was used to span the circumference C of the trunk.
This would give the diameter D of the trunk as D = C/π. In principle the area could then be calculated by

Exact: A = π (½D)2 = ¼C2/π ~ 0.0796 C2

Long before Coggeshall's time, a rule-of-thumb was in general use where a quarter of the circumference (as measured by the string around the trunk) determined the area using the formula

Rule-of-thumb: A = (¼C)2 = 0.0625 C2

The quarter-length of the string was actually assumed to represent the side of a square with the same area as the circular trunk's cross-section [9], [10]. Comparing the exact formula with the rule-of-thumb approximation shows that this practice resulted in a value for area (and therefore volume) that was about 21 % smaller than the exact value!
Other practices have been devised (often with better approximations) but the quarter-length method was preferred, probably because quartering a length of string (by halving two times) is easily accomplished. Another reason would be that sawing complete planks from a trunk resulted in a loss of wood comparable to the error in the quarter-length method (although that would vary with the dimensions of the plank's cross-section).
To calculate on Ayres' calculating disc the volume V of a trunk (in cubic feet) with a length of L feet and a circumference of C inches, we use the circumference in feet (C/12) and write:

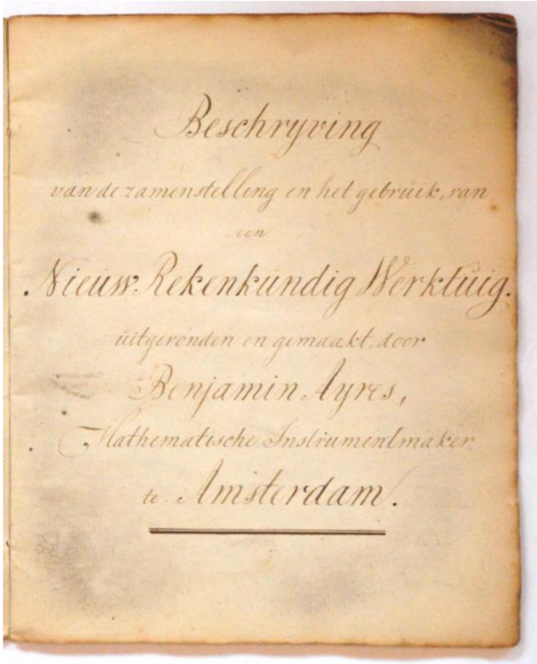
V = L A = L (¼C/12)2

This can be written in terms of proportions, as usual at the time:

V : L = (¼C)2 : (12)2

If we move the pointers 1 and 2 on the Girt Line (G) to the measured value ¼C and the inch-feet conversion gauge mark 12 respectively, we have the right side proportion value between the hairlines, but without the squaring yet. We square by using this proportion on the N-scale which represents the square power of the

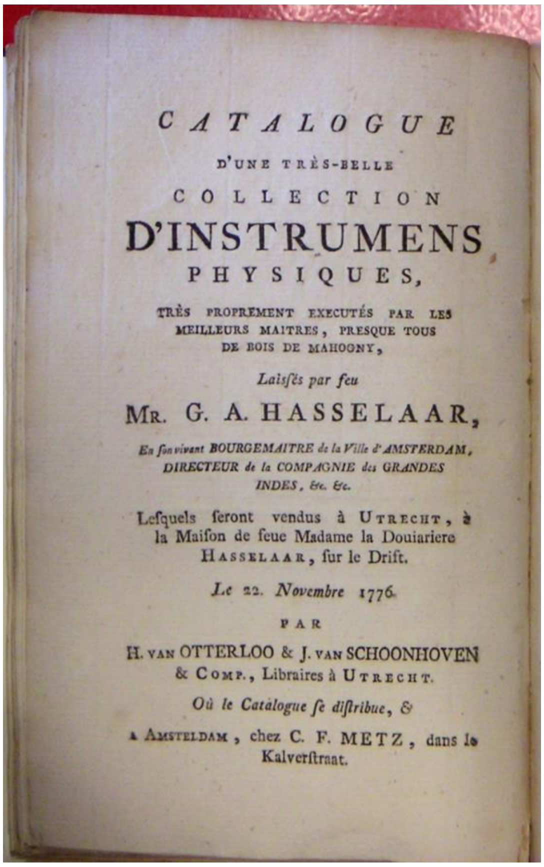
Girt scale. If we move the combination of pointers to the situation where pointer 2 on the N-scale points to L, we can read the value V on the N-scale under pointer 1.
This procedure is fully equivalent to using on a straight Coggeshall rule the Num scale (C) and the Girt scale (D). The conversion value "12" is often available as a Girt scale's gauge mark of a Coggeshall rule too. The apparent difference of the Coggeshall D-scale being shifted by 4 (running from 4 to 40), is not significant because it does not influence the calculation (the shift by 4 was probably done to get the gauge mark 12 in a comfortable mid-length position on the straight rule).



Beschryving
van de zamenstelling en het gebruik, van
een
Nieuw Rekenkundig Werktuig.
uitgevonden en gemaakt, door
Benjamin Ayres,
Mathematische Instrumentmaker
te Amsterdam.

Figure 4. Dutch Description by Ayres

Descriptive Document [4]
The sixteen-page Dutch description and manual for the Ayres calculating tool (also part of the UMU collection, and titled "Nieuw Rekenkundig Werktuig", see figure 4), matches the Ayres disc regarding its construction, but deviates slightly as it describes in total 9 scales, leaving out the single-decade Girt scales, and adding the goniometric scales expressed in Hours (for sundials) and Rhumbs (for compass points in navigation at sea). It is unfortunate that this document does not describe the intended use of inverted scales nor girt line.
Another deviation is that the manual gives as diameter options the values of 7, 8 or 9 "duimen" (Amsterdam inches: 1 duim = 2.573 cm), while the diameter of the calculating disc ME-32 is 15.5 cm, about 6 duimen. These deviations suggest that Ayres had designed more versions and may even have produced more specimens of this type of disc; however no other specimen is known to exist.
On the other hand, the description is a manuscript, not printed, and that might imply there is only one copy, or a few at most.



CATALOGUE
D'UNE TRÈS-BELLE
COLLECTION
D'INSTRUMENS
PHYSIQUES,
TRÈS PROPREMENT EXECUTÉS PAR LES MEILLEURS MAITRES, PRESQUE TOUS DE BOIS DE MAHOGNY,
Laissés par feu
MR. G. A. HASSELAAR,
En son vivant BOURGEMAITRE de la Ville d'AMSTERDAM, DIRECTEUR de la COMPAGNIE des GRANDES INDES, &c. &c.
Lesquels seront vendus à UTRECHT, à la Maison de feue Madame la Douairiere HASSELAAR, sur le Drift.
Le 22. Novembre 1776.
PAR
H. VAN OTTERLOO & J. VAN SCHOONHOVEN & COMP., Libraires à UTRECHT.
Où le Catalogue se distribue, &
A AMSTELDAM, chez C. F. METZ, dans le Kalverstraat.

Figure 5. Auction Catalog

Provenance [12]
The Ayres disc was part of the Hasselaer collection.
Gerard Aernout Hasselaer (1698-1766) was an exponent of the ruling classes in the Netherlands of the 18th century. He had been governor of various institutions in Amsterdam, but his career culminated in 1738 when he was appointed Director of the Dutch East India Company.
Later he became mayor of Amsterdam. Wealthy by birth and by marriage, he invested a great deal of time and money in collecting scientific instruments.
After he died, his widow moved -with his private "Instrument Cabinet"- to Utrecht. After she died in 1776, all instruments in the collection came up for auction on November 22d, 1776.
At that time the University of Utrecht already owned a modest collection of 66 scientific instruments. Professor Rossijn attended the Hasselaer auction [5] and bought on behalf of the University of Utrecht 23 instruments out of the more than 200 instruments offered. The Ayres calculating disc (auction item nr. 13)), together with a matching description booklet, was one of those acquisitions, see figures 5 & 6.

13 Een ronde koperen Plaat, zeer accuraat en fyn verdeeld, met twee Liniaaltjes voor de verdeeling der Sinus, Tangens en Secans, in een Mahogny houten kistje, met eene beschryving door *B. Ayrez.*

13 *Une plaque de cuivre a deux alidades pour trouver les Sinus, Tangens & Secantes; de toutes sortes de rayons fort beau, dans une caisse de Mahogny, par* B. Ayres.

Figure 6. Description of auction item nr. 13

Benjamin Ayres in Amsterdam [7], [8]

In London, 1724, the mathematical instrument maker Jonathan Sisson took as apprentice Benjamin Ayres at the age of about fourteen. In 1731, when Ayres completed his apprenticeship, he married Anne Sisson (who probably was Jonathan's sister). The couple moved in 1743 to Amsterdam because in that city there appeared to be a shortage of instrument makers.
Ayres made instruments for sea navigation, mainly compasses and octants, some of which have been preserved in Dutch science museums.
In 1749, Ayres was appointed Mathematical Instrument Maker to His Highness Prince Willem IV of Orange, Stadhouder of the Netherlands.
In that same year, at the intercession of the mathematician Cornelis Douwes, he was appointed instrument-maker in general to the Admiralties and the East and West India Companies. His tasks consisted of training a number of apprentice instrument makers, and producing instruments -mainly octants- for his employers.
In 1752 his career took a downward turn when his employment was ended prematurely because of his disappointing performance. Afterwards he left the Netherlands while his wife moved to The Hague and stayed there until her death in 1785. Much of Benjamin Ayres' life is still unknown, and dates are uncertain – from birth to death, and most events in between.

Attribution

As Ayres' signature is not inscribed on the disc, we need further proof of its attribution. The description document that was auctioned together with the calculating disc, clearly states Ayres as the maker of the described instrument, see figure 4. The description of the construction -and of most of the scales- fits the UMU object well (though not fully), and can be considered sufficient proof of attribution. The fact that the description was in the Dutch language suggests that it was made in Holland, after Ayres moved to Amsterdam.
Another interesting issue is the origin of Ayres' knowledge of logarithmic scales and calculating discs. He probably acquired that knowledge in his London period, but there were not that many circular slide rules known at the time. Oughtred's calculating disc (1620s) was of course an early example for Ayres, and maybe also a disc with spiraling long scales in the Science Museum, London, that is attributed to John Brown (~1670), see [6, pp. 15-16], and [11]. But Ayres' scales were sufficiently different from those early examples, and can therefore not be called a copy. Also he improved the classical "knife-edge" pointers by adding hairlines.
Another example for him may have been the calculating disc by Benjamin Scott, who published a description [3] in 1733 (while Ayres was in London at Sisson's, close to Scott's workshop - on the London Strand too); Scott's publication did not include a picture, nor details of construction. His mathematical scales were less advanced than those of Ayres, covering only the Num, Sin and lower Tan scales, but Scott had included a large number (17) of specialized astronomical scales.
A later circular slide rule, described by Nicholson in 1797 [6, pp. 36-37], shows pointers with metal hairlines too, closely following Ayres' example.
Ayres' knowledge about timber calculations must have come from publications such as the 1729 "Carpenter Rule" pamphlet by Henry Coggeshall [2], or general books on measuring, e.g. [1].

Conclusions

The calculating disc, described in this article, is almost certainly made by Ayres in Amsterdam around 1750, considering the matching description that accompanied the disc at the Hasselaer auction. The design of the disc was innovative because Ayres used for the first time metal hairlines on the rotating pointers. He also introduced inverted scales on a circular slide rule, and a circular girt line with the gauge mark "12".

Acknowledgments

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Doos met draaibare Neperstaafjes, aanwezig in het CNAM-museum te Parijs