26. Renard's Preferred Numbers

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A concise survey of the history, theory and applications of preferred numbers, with reference to Aristo and Nestler slide rules

Summary

Most slide rule collectors have in their collection Aristo- or Nestler- slide rules with on the rear side tables with R5-, R10-, R20-, E6- and E12-series. Often they possess plastic strips from the Aristo-firm with gauge points that denote the so-called preferred numbers.

The French lieutenant Charles Renard discovered preferred numbers in 1877. He found that it is not necessary to produce all values of an item's parameter when a large number of different values is required. A small series of standard values, a series of preferred numbers, is sufficient to cover all needed values. Consequently, production and logistical costs can be reduced enormously.

The concept of tolerance forms the hart of the preferred number theory. The mathematics of preferred numbers, which in fact is calculus of geometric series, covers this concept, as well as the notion of the number of significant digits that is applied to write the preferred values.

Because of the logarithmic nature of slide rules, preferred numbers on C- and D-scales can easily be correlated with easy-to-find and –remember values on the mantissa-scale L of slide rules. That's why it is not very difficult to calculate with the Renard-numbers on slide rules.

Introduction: Aristo NZ-rules

In August 2003, members of the International Slide Rule Group (ISRG) deeply discussed *preferred numbers*, numbers of preference that are called *Normzahlen* (NZ) in German. Other names for these numbers are *normal numbers, Normungszahlen or Renard-numbers* (R-numbers).

The discussion was motivated by scales and certain gauge points on the Aristo-1364, a thin, cheaply constructed plastic NZ-rule (in fact a plastic strip) that Aristo delivered with some slide rules. The Aristo-89-Rietz slide rule had an NZ-scale on the rear side and was therefore called Aristo 89NZ in the brochures. (See match-number 106 in Herman van Herwijnen's Blue Book). See figures 1 and 3.

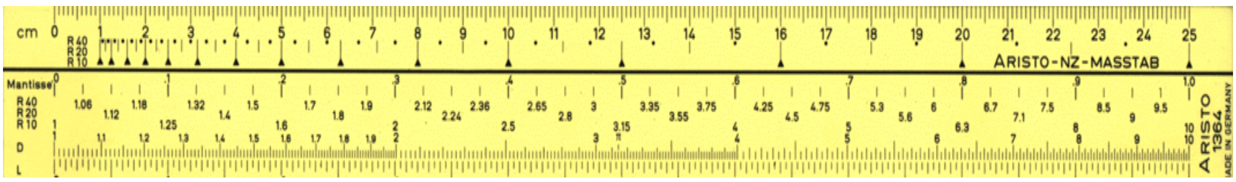

Figure 1: The Aristo 1364

See also reference [10].

Visits on the Internet showed the Aristo 1364 is not the only NZ-rule Aristo produced. Michael Gährken's website (See [1]) shows a picture of the Aristo 1367, a rule that Aristo produced in combination with the Aristo 0968 at the beginning of the 1960's. See figure 2.

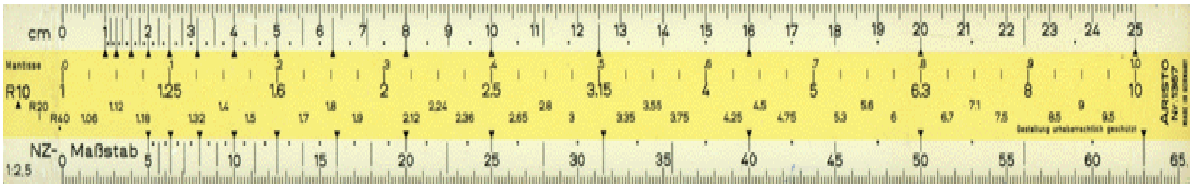

Figure 2: The Aristo 1367

According the Blue Book the Aristo 0968 was sold in combination with another NZ-rule.
Concerning match-number 361 the Blue Book mentions an Aristo-NZ-rule with the number 1397. Though one number further, under match-number 362 the Blue Book mentions an Aristo 0968 in combination with a NZ-rule that bears the product number 1364.

The Aristo NZ-rules have scales, which are denoted by R10, R20 en R40. These scales are called *Renard-scales*.

Besides rules with NZ-scale divisions according preferred numbers, there exist also Nestler slide rules that bear on the rear side tables with R5-, R10-, R20- and/or R40-numbers. In addition, can one find on the rear side of slide rules tables with the E6-, E12- en E24-series. See [10]. Sometimes only one series of normal numbers is mentioned. My own collection contains a Nestler 21, Darmstadt, of which the rear side shows the series of numbers 10; 12.5; 16; 20; 25; 31.5; 40; 50; 63; 80, hence numbers that belong to the R10-series. By the way: Nestler uses the name *Normungszahlen* on this slide rule, in stead of the more usual Normzahlen. See figure 4. My collection contains a Nestler Multimath-Duplex, no. 0130, of which the case contains also a plastic strip, without any product number, with on one side the series R5, R10, R20 en R40 in 3 significant decimal figures and on the other side the series E6, E12 en E24 in 2 significant figures.

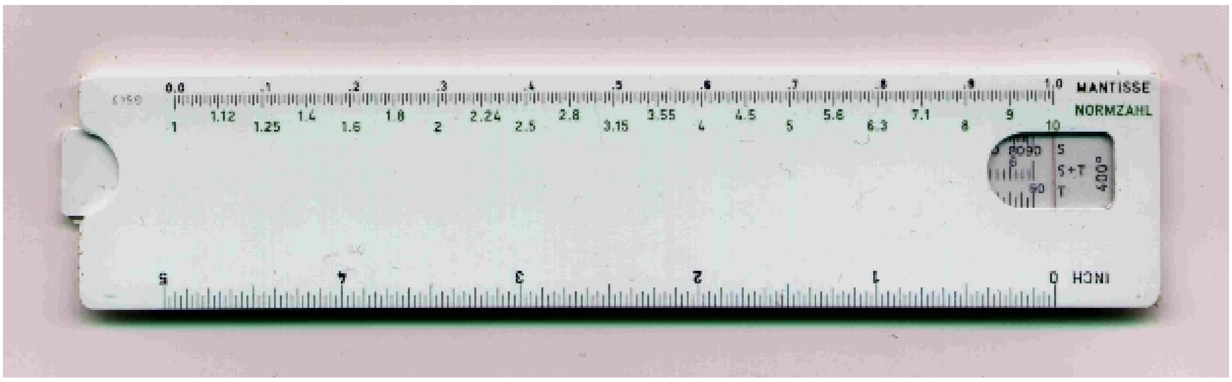

Figure 3: Rear side of an Aristo 89NZ

Charles Renard

Give an electronics engineer the following series of numbers: 10, 12, 15, 18, 22, 27, 33, 39, 47, 56, 68, 82, 100 and he will tell you immediately that these numbers belong to the E12-series, a series that is used for instance for the values of electric resistors with a tolerance of 10%. If you wish he would tell you that there also exists E24-, E48- en E96-series and that those series are applied, dependent upon the necessary tolerance of the components in use. See [6].

Give a mechanics engineer one of the following series 100, 160, 250, 400, 630, 100 or 112, 180, 280, 450, 710 and 1120 and he will explain you those series are applied for the number of revolutions of tooling instruments. See [5]. One can find other series also. For instance the smaller types of drilling-machines, of which the velocity is adjusted in discrete steps, have the following series of numbers of revolutions per minute: 1000, 1400, 2000, 2800, 4000, 5600, 8000, 11200.

The examples above mentioned all make use of a *Renard-series* of numbers, named after the French army captain *Charles Renard* (1847-1905), who professionally worked with big balloons and primitive air ships. The French army used balloons to observe enemy lines. Per balloon no less than 400 ropes were needed, but in the year 1877 Charles Renard discovered how to reduce this enormous number to a mere 17 standard ropes that could be used for all applications.

Renard discovered that it is pointless to apply specific values that lie within the tolerance area of a standard value. It is always possible to round off a specific value to above or below to reach a certain standard value, because of which the number of necessary different values of a parameter can be reduced enormously. The standard values are roundings off of so-called *Renard-numbers*.

Charles Renard: b. 1847, France, d. 1905, France, French military engineer, chief builder of the first true dirigible; i.e., an airship that could be steered in any direction irrespective of wind and could return under its own power to its point of departure. In 1884 Renard and Arthur Krebs, French Army captains at the Aérostation Militaire, Chalais-Meudon, completed the dirigible "La France," which on August 9 of that year made its first flight, a circular journey of 7 or 8 kilometres (about 4 to 5 miles). Earlier (1871) Renard had flown a pilotless heavier-than-air craft, a 10-winged model glider.

In general by the restriction to standard values, a substantial reduction in production and logistical costs is possible. For instance, a certain ISO-norm for small bolts gives *first preference* for bolts M4, M6, M10, M16 and M24. The *second preference* consists of the bolts M3, M5, M8, M12, M20 and M30, while only in exceptional cases the application of bolts M3.5, M4.5, M14, M18, M22 and M27 is justified. The ISO-norm calls this last mentioned series *third preference*. Notice: for example, bolts M7 don't belong to this series.

If desired, the series could be extended below or above, by repeating the same pattern. These means manufacturers, by preference over the whole world, try to construct their products by using only bolts of the first preference. It is clear such a preference for certain standard values makes a simple exchange of products possible and an enormous reduction of production costs. That is why the standard numbers 4, 6, 10, 16 en 24, for example in use for metal bolts, are called *preferred numbers* (Normzahlen, Normungszahlen, NZ-numbers). You can also observe the classification in three series of preferred numbers in the hardware shops. In the cheapest shop, you can only buy bolts of the first preference. Probably the better shop can deliver also bolts of the second preference, while only the shop of highest quality can deliver you bolts of the third preference.
In addition, what about bolts M7? The manufacturer, who wants to apply them really, would have to produce them himself or will have to pay a substantial price for them. However, French car manufacturers like Citroën, but even German BMW use them, despite these facts.

Renard-numbers

The numbers of Renard form a geometric series Rn with common ratio $\alpha = \sqrt[n]{10}$. Per definition, the number 1 is a Renard-number. The number n is called *power number* of the series.

Mostly a Renard-series (strictly speaking this is always a partial series) starts with the number 1 or with the number 10, but that is by no means necessary. For instance, in the mechanics engineering, we can find series that begin with the number 40. The geometric series is, to the left as well as to the right, infinitely long. However, to characterise a Renard-series it is sufficient to mention a sequence of n subsequent numbers.

Table 1	1,000
	$\alpha = \sqrt[5]{10} = 1.585$
	$\alpha^2 = \left(\sqrt[5]{10}\right)^2 = 2.512$
	$\alpha^3 = \left(\sqrt[5]{10}\right)^3 = 3.981$
	$\alpha^4 = \left(\sqrt[4]{10}\right)^4 = 6.310$
	$\alpha^5 = \left(\sqrt[5]{10}\right)^5 = 10.00$

For example, n equals 5 and hence $\alpha = \sqrt[5]{10}$, then we find the geometric series of which table 1 gives 5 subsequent values. Notice that the numbers in this table are written in 4 significant decimal figures.

In table 2, the Renard-numbers for power numbers n = 5 up to n = 12, and for power numbers n = 20, n = 24 and n = 40 are shown. The Renard-series in table 2 all begin with the value 10.
If we round off the values of the R5-series to 1 decimal figure after the decimal point and if we multiply every number in the series by 10, then we find a well known series of preferred numbers: R5 = 10; 16; 25; 40; 63; 100. This series of rounded Renard-numbers is used

for the focal lengths of lenses and for maximum allowable voltage of capacitors.

Table 2a: Renard-numbers from 10 up to 100 for n = 5 upto n = 12							
5	**6**	**7**	**8**	**9**	**10**	**11**	**12**
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
15.85	14.68	13.89	13.34	12.92	12.59	12.33	12.12
25.12	21.54	19.31	17.78	16.68	15.85	15.20	14.68
39.81	31.62	26.83	23.71	21.54	19.95	18.74	17.78
63.10	46.42	37.28	31.62	27.83	25.12	23.10	21.54
100.00	68.13	51.79	42.17	35.94	31.62	28.48	26.10
	100.00	71.97	56.23	46.42	39.81	35.11	31.62
		100.00	74.99	59.95	50.12	43.29	38.31
			100.00	77.43	63.10	53.37	46.42
				100.00	79.43	65.79	56.23
					100.00	81.11	68.13
						100.00	82.54
							100.00

Some series of rounded Renard-numbers form Renard-series of *standardised preferred numbers.* With the notation R5 the original, Renard-series of not rounded values is indicated as well as the matching series of preferred numbers. Sometimes the rounding off to preferred numbers has been done in a “strange manner” because of certain practical considerations. In table, 3 we see the mostly applied standardised preferred numbers. The series that are denoted with E are used in electronics.

Notice that several numbers of the E12-series are integer numbers that are formed by using rounding off the values of the R12-series. When we study the E12-series, we mention that sometimes the applied rounding differs more or less strongly from the usual rounding. For instance, we see the value 31.62 in R12, while we find the number 33 in the E12-series.

The above mentioned shows multiplication by a power of 10 of every member of a (partial) Renard-series results in another part of the same series. Thus, it doesn’t matter with which number the series starts.

Notice that the power number n equals the number of parts in which every decade is divided.

The ISO, the international organisation for standardising, gives in her normalisations 4 standardised Renard-series of preferred numbers: the series that are numbered R5, R10, and R20 in table 3, and the series R40 = 10; 10.6; 11.2; 11.8; 12.5; 13.2; 14; 15; 16; 17; 18; 19; 20; 21.2; 22.4; 23.6; 25; 26.5; 28; 30; 31.5; 33.5; 35.5; 37.5; 40; 42.5; 45; 47.5; 50; 53; 56; 60; 63; 67; 71; 75; 80; 85; 90; 95; 100. Due to its considerable length, it is not mentioned in table 3.

Remarkable enough the standardised preferred numbers in table 3 are written with a different number of decimal figures!

Table 2b: Renard-numbers for n = 20, 24, 40			
20	**24**	**40** **1st half**	**40** **2nd half**
10.00	10.00	10.00	
11.22	11.01	10.59	33.50
12.59	12.12	11.22	35.48
14.13	13.34	11.89	37.58
15.85	14.68	12.59	39.81
17.78	16.16	13.34	42.17
19.95	17.78	14.13	44.67
22.39	19.57	14.96	47.32
25.12	21.54	15.85	50.12
28.18	23.71	16.79	53.09
31.62	26.10	17.78	56.23
35.48	28.73	18.84	59.57
39.81	31.62	19.95	63.10
44.67	34.81	21.13	66.83
50.12	38.31	22.39	70.79
56.23	42.17	23.71	74.99
63.10	46.42	25.12	79.43
70.79	51.09	26.61	84.14
79.43	56.23	28.18	89.13
89.13	61.90	29.85	94.41
100.00	68.13	31.62	100.00
	74.99		
	82.54		
	90.85		
	100.00		

Besides the 4 mentioned R-series there are series that are denoted by E: the E6-, E12-, E24-, E48- and E96-series that are used in the electro-technics, in particular electronics.

Table 3 only mentions the E6- and the E12-series. The E24-series, that is not mentioned in table 3, is: 10; 11; 12; 13; 15; 16; 18; 20; 22; 24; 27; 30; 33; 36; 39; 43; 47; 51; 56; 62; 68; 75; 82; 91; 100.

Besides the primary series of standardised preferred numbers, like the mentioned values in table 3, we distinguish also *standardised singularly derived R-series*, which are derived from the primary series by means of extra rounding off:
R′10 = 10; 12.5; 16; 20; 25; 32; 40; 50; 63; 80; 100.
R′20 = 10; 11; 12.5; 14; 16; 18; 20; 22; 25; 28; 32; 36; 40; 45; 50; 56; 63; 71; 80; 90; 100.
R′40 =10; 10.5; 11; 12; 12.5; 13; 14; 15; 16; 17; 18; 19; 20; 21; 22; 24; 25; 26; 28; 30; 32; 34; 36; 38; 40; 42; 45; 48; 50; 53; 56; 60; 63; 67; 71; 75; 80; 85; 90; 95; 100.

Table 3: Mostly used standardised preferred numbers

	R5	**E6**	**R10**	**E12**	**R20**
n =	5	6	10	12	20
α =	**1.585**	**1.468**	**1.259**	**1.212**	**1.122**
Begin:	10	10	10	10	10
	16	15	12.5	12	11.2
	25	22	16	15	12.5
	40	33	20	18	14
	63	47	25	22	16
	100	68	31.5	27	18
		100	40	33	20
			50	39	22.4
			63	47	25
			80	56	28
			100	68	31.5
				82	35.5
				100	40
					45
					50
					56
					63
					71
					80
					90
					100

The *standardised doubly derived series of preferred numbers are:*
R″5 =10; 15; 25; 40; 60; 100.
R″10 = 10; 12; 15; 20; 25; 30; 40; 50; 60; 80; 100.
R″20 = 10; 11; 12; 14; 16; 18; 20; 22; 25; 28; 30; 35; 40; 45; 50; 50; 55; 60; 70; 80; 90; 100.

Partial series of standardised Renard-series are for instance denoted by R″20 (18.55). In this description the begin value and end value belong to the partial series.

Partial series that consist of for example every third item of the R40-series, that starts with the number 10 and ends with the value 95, are written as:

R40/3 (10..95) = 10; 11.8; 14; 17; 20; 23.6; 28; 33.5; 40; 47.5; 56; 67; 80; 95.

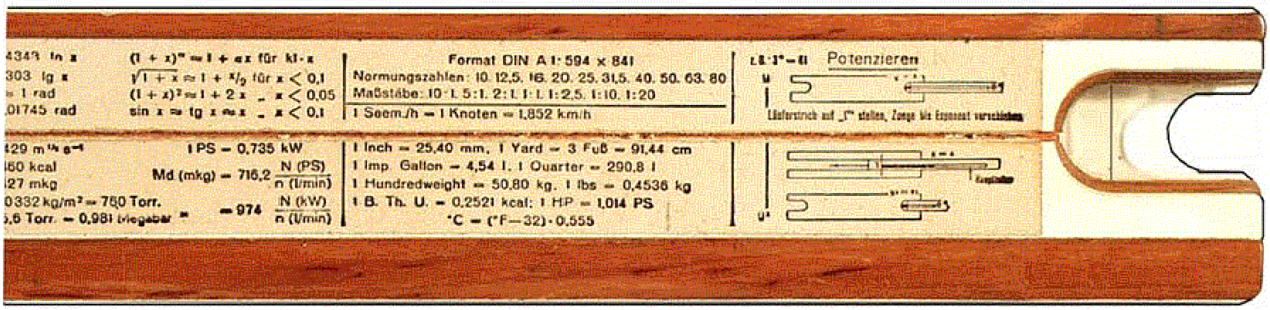

Figure 4: Rear side of the Nestler 21, with Normungszahlen from the R10-series

Tolerance circles

We study a Renard-series of n+1 subsequent numbers that starts with 1 and has common ratio $\alpha = \sqrt[n]{10}$:

$$g_0 = \alpha^0 = 1;\ g_1 = \alpha^1;\ \ldots..;\ g_{k-1} = \alpha^{k-1}\ ;\ g_k = \alpha^k\ ;\ \ldots..\ ;\ g_n = \alpha^n = 10.$$

We call the interval of real numbers from 1 to 10 *basic decade.* Slide rule users would say the basic decade is the C- or D-scale on slide rules. In figure 5 these numbers are shown, together with the corresponding *tolerance circles*. In order to make the picture more surveyable; the number axis has been duplicated. The numbers g_k are really positioned in the centre of the tolerance circles.

Every decade of the positive number axis is covered by n connecting circles. Notice that only the right half of the most left circle and the left half of the most right circle belong to the coverage of the decade.

The part of the number axis that lies within the circle is called the *tolerance area* or *tolerance interval* of the number in the centre of the corresponding circle.

Due to the geometrical character of the Renard-series, the radius of the kth circle is α times the radius of the (k-

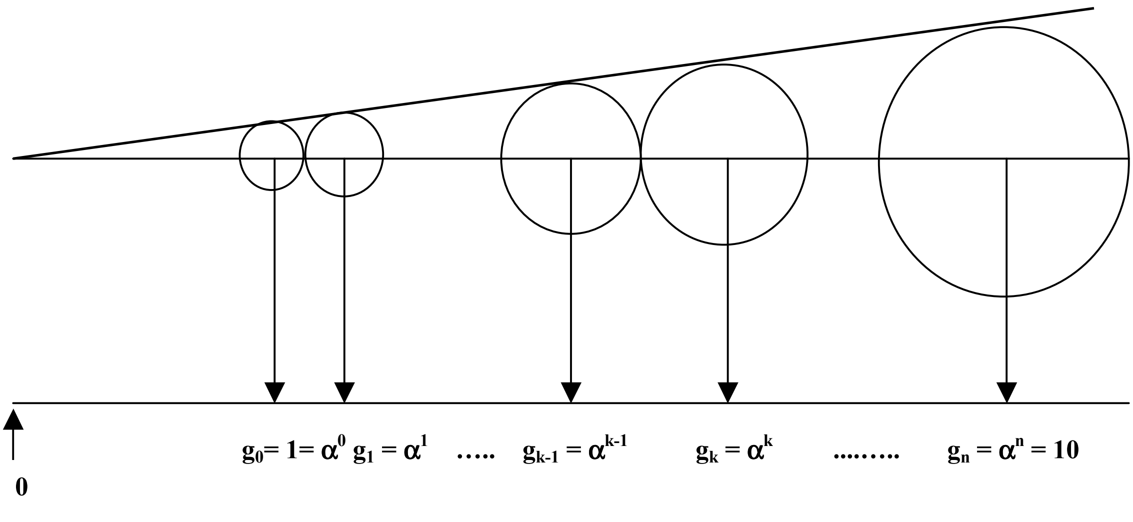

Figure 5

1)th circle left of it. Therefore, we find that the highest points of the tolerance circles lie exactly on a straight line through the origin of the number axis.

We denote the circles in figure 5, from left to right, by C_0 up to C_n.

The centre point of circle C_0 is the number 1; the centre of C_n is the value 10.

We denote an arbitrarily chosen circle by C_k. The radius of this circle C_k we denote by R_k. This radius R_k we call *tolerance radius* of C_k.

The length of the tolerance radius of tolerance circle C_k equals: $R_k = \alpha^k$. R0

From figure 5 we derive the following equation with a geometric series:

$$
\begin{aligned}
9 &= (1+\alpha)R_0 + \alpha(1+\alpha)R_0 + ... + \alpha^{n-1}(1+\alpha)R_0 \\
&= (1+\alpha)R_0 \cdot (1+\alpha+\alpha^2 + ... + \alpha^{n-1}) \\
&= (1+\alpha)R_0 \cdot \frac{\alpha^n - 1}{\alpha - 1} = (\alpha+1)R_0 \cdot \frac{9}{\alpha - 1}
\end{aligned}
$$

Hence, we find:

$$R_0 = \frac{\alpha - 1}{\alpha + 1} = \frac{\sqrt[n]{10} - 1}{\sqrt[n]{10} + 1}$$

For several values of the power number n, table 4 gives the length of radius R_0, thus the radius of the circle with the number 1 as centre point.

Table 4: Radius R_0 as a function of power number n

n =	5	6	10	12	20	24	40
α =	1.585	1.468	1.259	1.212	1.122	1.101	1.059
R_0 =	0.226	0.190	0.115	0.096	0.058	0.048	0.029

Tolerance and fractional uncertainty

In the previous paragraph, we saw the length of the tolerance radius of tolerance circle C_k equals:

$$R_k = \alpha^k . R_0$$

The meaning of tolerance radius of a parameter of an object, which has the *nominal value* $g_k = \alpha^k$, is that the true value can be situated anywhere in the interval:

$$\alpha^k - \alpha^k R_0 < g_k = \alpha^k < \alpha^k + \alpha^k R_0$$

Notice that this definition doesn't say anything about the probability distribution over the concerned interval. Thus, it says nothing about the probability of finding the true value in any specific part of the tolerance area. According to this definition, the true value can be situated at the far-left end of the interval as well as at the far right side of the interval or anywhere in the central part.

Hence, the *absolute uncertainty* (absolute error) in the nominal value g_k is:

$$\Delta(g_k) = \Delta(\alpha^k) = \alpha^k R_0$$

In practical situations, we are not too interested in the absolute uncertainty. We just use geometric series in order to say something about the relative uncertainty.

The *relative or fractional uncertainty* (relative error) equals the absolute uncertainty divided by the nominal value:

$$\delta(g_k) = \delta(\alpha^k) = \frac{\Delta(\alpha^k)}{\alpha^k} = \frac{\alpha^k R_0}{\alpha^k} = \frac{R_0}{1} = \frac{\alpha - 1}{\alpha + 1}$$

Of course, we can express the relative uncertainty in percentages. We then find the *percentage uncertainty* or tolerance:

$$\tau(g_k) = \delta(g_k) \cdot 100\% = \frac{R_0}{1} \cdot 100\% = \frac{\alpha - 1}{\alpha + 1} \cdot 100\%$$

Table 5 shows the same values as table 4, but with addition of the tolerance, as a function of the power number n.

Table 5: Tolerance (percentage uncertainty) as a function of power number n

n =	5	6	10	12	20	24	40
α =	1.585	1.468	1.259	1.212	1.122	1.101	1.059
R_0 =	0.226	0.190	0.115	0.096	0.058	0.048	0.029
$\tau(\alpha^k) =$	22.6%	19.0%	11.5%	9.6%	5.8%	4.8%	2.9%

We see the tolerance is a function of n, but not of k, so not of the precise place of a value on the number axis. For every Renard-number with the same power number n, the percentage uncertainty or the tolerance is identical. This is exactly the reason of existence for Renard-numbers.

Usually one rounds the tolerance values of the E-series off to easily remembered values. For the E6-, E12- en E24-series one uses the notations: ±20%, ± 10%, ±5% respectively. See the plastic strip with Normzahlen that was delivered with the Multimath-Duplex, Nestler 0130.

Nowadays in electronics, one can find applications of the E48- en E96-series. Their tolerances are easily to find. A doubling of the power number n approximately halves the tolerance: for the E48- and the E96-series we find approximate tolerances of ±2.5% en ±1.25% respectively. In daily practise manufacturers even use the more profound rounding off to 2% and 1%.

In the same manner we can round off the tolerances of other R-series from table 5 to easily remembered percentages. The tolerances of the R5-, the R10-, the R20- and the R40-series are respectively ± 24%, ±12%, ±6% and ± 3%. We choose these approximations in order that a doubling of the power number n halves the tolerance.

The significant decimal figures of preferred numbers

In table 3 we can see that for a constant value of the power number n the number of significant decimal figures of the preferred numbers vary. Most preferred numbers are described with 2 significant decimal figures, but some with 3. The R10-series of preferred numbers has the 3-figure values 12.5 and 31.5, while the other numbers are written in 2 significant figures.

This way of writing wrongly suggests that there is, in case of 3 significant decimal figures, a higher accuracy than in case of 2 significant figures. The plastic strip of the Nestler 0130 (Multimath-Duplex) gives all the preferred numbers of the series R5, R10, R20 and R40 in 3 significant decimal figures, while table 5 shows quite a variety of different tolerances (and uncertainties).
This confronts us with all kinds of questions about the accuracy of the standardised preferred numbers.

The relation between the number of significant figures (or decimal digits) in a value and the corresponding uncertainty is a source of many difficulties. Remarkably, enough, this relation between accuracy and the applied number of significant digits is not exactly defined. Not in mathematics, not in physics and not in technology. The definition depends upon the interpretation of the mathematician, engineer or scientist that is using numbers with a certain number of significant decimal figures.

When the value of a certain parameter equals for instance 6.74, we say that we use 3 *significant decimal figures* or 3 *significant decimal digits* in that value. When we write 6.740, we use 4 significant figures. In the last case, we mostly presume there is a greater accuracy than in the first case.

Leading zeroes in a number are not significant. When we write 0.0635 then we write the number in 3 significant decimal figures, although we use more than 3 decimals.

Most easily, we find the number of significant figures when we write the number in *scientific notation*.
The number 0.0635 we can write as 6.35×10^{-2}. In relation with calculating on a slide rule, we use mantissas in the interval from 1 to 10, but this is not necessary. However, because most calculations on a slide rule are performed in connection with the D-scale, our choice of mantissas between 1 and 10 is natural.

The scientific notation shows very well why leading zeroes are not significant but that trailing zeroes are significant. 6.74×10^3 is not the same as 6.7400×10^3. Obviously, we presume a greater precision in the last number.

As said above there is not one uniform definition of accuracy in connection with the number of significant decimal figures. Different users have different interpretations. The mathematician would say that the uncertainty is half the place value of the last decimal figure of the mantissa, multiplied by a power of 10. He reads 6.74×10^3 as $(6.74 \pm \frac{1}{2} \times 10^{-2}) \times 10^3$ or as $(6.74 \pm 0.005) \times 10^3$.

The electronics engineer who for example reads electrical voltage from an analogue voltmeter, and who rounds the shown value off to the *nearest dash* must take into account an error of "half a unity". Because of other random errors of the measuring instrument, the total uncertainty is usually higher then this half of a unity.

In addition, when a digital voltmeter is used the error of measurement is in general higher than half the place value of the last figure in the mantissa, multiplied by a power of 10.

Empirical results of measurements are written as for instance $(6.74 \pm 0.01) \times 10^3$, as $(6.74 \pm 0.05) \times 10^3$ or, in exceptional case even as $(6.74 \pm 0.23) \times 10^3$. Obviously, the number of significant decimal figures in the notation of a number means nothing more than just the number of figures in the mantissa.

Only in case of very accurate measurements, it is meaningful to give the uncertainty in 2 decimal figures. Writing the result of a measurement as 6.74 ± 0.02567 is incredible. In almost all practical situations, the uncertainty of a measured value must be written in only 1significant decimal figure. (Nevertheless, when the first digit of the uncertainty is 1 or 2, one could decide to use two significant digits in the uncertainty.)

Instead of 6.74 ± 0.02567 we write 6.75 ± 0.03. The last significant digit in the mantissa of the result of a measurement must have the *same order of magnitude* (= has the same decimal position) as the uncertainty.

Hence, in practical engineering it is not very meaningful to use the rather strict mathematical definition of uncertainty in a number that is written in n significant decimal figures.

In case of empirical observations that are written with a mantissa with n significant digits and of which nothing is mentioned about the accuracy, one *usually* says the uncertainty equals 10^{-n+1}. Of course, this is not a proper definition but a rule of thumb that nevertheless holds in a relatively extensive category of practical cases. Hence, that uncertainty equals the place value of the last decimal figure of the mantissa multiplied by an appropriate power of 10. So when the value is denoted as 6.74 (thus when noting is said about the uncertainty in the number), we usually understand this number as: 6.74 ± 0.01.
When we apply this rule of thumb definition for uncertainty in a number that is denoted in n significant decimal figures, we can easily see a practical relation between the number of significant digits in a value and the relative uncertainty or tolerance.
See table 6.

From this table, we can conclude that in every day engineering practice mostly we don't need more significant digits than 2 and that more than 3 digits are seldomly required.

By the way: the slide rule is perfectly adapted to this particular interpretation of uncertainty in relation to the number of significant figures. When calculating on a slide rule we seldomly can read more than 3 significant figures. When we use an electronic calculator, we risk using more significant digits than is meaningful.

For instance it is often difficult to accept that writing the surface area of a small plate of wood with a length of 6,8 [dm] and a height of 7.3 [dm] as 49.64 [dm^2] is not very meaningful, while the calculation has been done with so much accuracy. Mention how the do-it-your-self-shop exactly knows how to reduce those 4 digits!

Table 6: Practical relation between the number of significant decimal digits in a value and percentage uncertainty or tolerance		
	Percentage uncertainty or tolerance	
Number of significant digits	**Between**	**Roughly**
1	10% en 100%	50%
2	1% en 10%	5%
3	0.1% en 1%	0.5%
4	001% en 0.1%	0.05%

Therefore, by interpreting the meaning of significant figures one has to be very cautious. The preferred number 12.5 in the R10-series means 12.5 ± 1.4; the preferred number 31.5 in that series can best be understood as 31.5 ± 0.4. Here we see much higher uncertainties than the place value of the last digit of the mantissa multiplied by 10!

Because of the values in table, 6 preferred numbers should be written in only 2 significant numbers. This preference for 2 significant digits forms the background of the need for the derived series of standardised preferred numbers. However, sometimes it is necessary to use 3 significant digits in preferred numbers in order to get a good spreading of the numbers in a decade, especially for numbers between 1 and 2.

One doesn't use 3 significant digits to denote a higher accuracy! For example, in electronics the E48-series is often used for resistors with a tolerance of 2%. (In fact, the E48-series has 2.4% as tolerance). Following table 6 we should use 2 significant decimal figures, but that would mean we couldn't make any difference between the numbers 1.27 and 1.33, when both would be rounded off to 1.3.

Preferred numbers and slide rules

Again, we study a Renard-series of n + 1 subsequent numbers, with 1 as starting number and common ratio $\alpha = \sqrt[n]{10}$:

$$g_0 = \alpha^0 = 1;\ g_1 = \alpha^1;\ \ldots;\ g_{k-1} = \alpha^{k-1};\ g_k = \alpha^k;\ \ldots;\ g_n = \alpha^n = 10.$$

Because Renard-numbers are defined by means of a power root of 10, there exists a simple relation between Renard-numbers and their logarithms. Table 7 gives the logarithms in decimals and in fractions of the preferred numbers of the R5-, R10-, R20- and R40-series. The logarithms are chosen in the interval from 1 to 10, so in fact the logarithms in this table are identical with the mantissas.

Table 7: Preferred numbers and their logarithms								
$\log(g_{n;k})$	**k/40**	**R40**	**k/20**	**R20**	**k/10**	**R10**	**k/5**	**R5**
0.000	0/40	1.00	0/20	1.00	0/10	1.00	0/5	1.00
0.025	1/40	1.06						
0.050	2/40	1.12	1/20	1.12				
0.075	3/40	1.18						
0.100	4/40	1.25	2/20	1.25	1/10	1.25		
0.125	5/40	1.32						
0.150	6/40	1.40	3/20	1.40				
0.175	7/40	1.50						
0.200	8/40	1.60	4/20	1.60	2/10	1.60	1/5	1.60
0.225	9/40	1.70						
0.250	10/40	1.80	5/20	1.80				
0.275	11/40	1.90						
0.300	12/40	2.00	6/20	2.00	3/10	2.00		
0.325	13/40	2.12						
0.350	14/40	2.24	7/20	2.24				
0.375	15/40	2.36						
0.400	16/40	2.50	8/20	2.50	4/10	2.50	2/5	2.50
0.425	17/40	2.65						
0.450	18/40	2.80	9/20	2.80				
0.475	19/40	3.00						
0.500	20/40	3.15	10/20	3.15	5/10	3.15		
0.525	21/40	3.35						
0.550	22/40	3.55	11/20	3.55				
0.575	23/40	3.75						
0.600	24/40	4.00	12/20	4.00	6/10	4.00	3/5	4.00
0.625	25/40	4.25						
0.650	26/40	4.50	13/20	4.50				
0.675	27/40	4.75						
0.700	28/40	5.00	14/20	5.00	7/10	5.00		
0.725	29/40	5.30						
0.750	30/40	5.60	15/20	5.60				
0.775	31/40	6.00						
0.800	32/40	6.30	16/20	6.30	8/10	6.30	4/5	6.30
0.825	33/40	6.70						
0.850	34/40	7.10	17/20	7.10				
0.875	35/40	7.50						
0.900	36/40	8.00	18/20	8.00	9/10	8.00		
0.925	37/40	8.50						
0.950	38/40	9.00	19/20	9.00				
0.975	39/40	9.50						
1.000	40/40	10.00	20/20	10.00	10/10	10.00	5/5	10.00

In this paragraph, it is useful to enhance the notation of Renard-numbers. In stead of g_k, in the decade from 1 to 10, we shall write $g_{n;k}$, in order to show that Renard-numbers are a function of n as well as of k.

We can read $g_{n;k}$ as the k[th] Renard-number in the decade from 1 to 10, that belongs to the Rn-series. Also the rounding off of the Renard-number to a standardised preferred number we denote by $g_{n;k}$.

We find as logarithm of $g_{n'k}$:

$$\log(g_{n;k}) = \log(\alpha^k) = k \cdot \log(\alpha) = k \cdot \log(\sqrt[n]{10}) = \frac{k}{n}$$

In this formula and next formulae the log's base is 10, so in Dutch and American notation:

$$\log(x) = {}^{10}\log(x) = \log_{10}(x)$$

De numbers (the mantissas) k/n are situated on the L-scale of a slide rule and can be found easily because of the linearly divided marks on this scale.

The corresponding Renard-number:

$$g_{n;k} = 10^{k/n}$$

(and when we take the sometimes unusual rounding off not too problematically, even the corresponding standardised preferred number) we read on the D-scale.

Suppose we like to know the 3rd number of the R40-series, than we look for 3/40 on the L-scale (which is quite easy) and read on the D-scale the corresponding preferred number 1.18. Notice the number 1 we call the 0th number of the Renard-series!

In the same manner, we find the 14th number of series R20 by looking for 14/20 on the L-scale and reading the corresponding preferred number on the D-scale. We find 0.50. Because 14/20 equals 7/10, we immediately see that the number 0.50 is also a number of the R10-series. (Moreover, we conclude from the notation 7/10 for the mantissa, it is the 7th value in series R10.)

The logarithm of the 32nd number in the R40-series equals 32/40, that can be written as 16/20, and as 8/10 and as 4/5. We conclude the preferred number 6.30 belongs to the R5-, R10-, R20- and R40-series.

Multiplication and division of preferred numbers

Because of the nature of Renard-numbers, the product and the quotient of two of those type numbers give a Renard-number as a result:

$$\log(g_{n;k} \cdot g_{n;l}) = \log(g_{n;k}) + \log(g_{n;l}) = \frac{k+l}{n}$$

If the sum k + l is more than n, it is not possible to read the corresponding Renard-number from the D-scale of a slide rule. The D-scale is just not long enough. However, after n Renard-numbers the series repeats itself, but with numbers that are 10 times the numbers in the basic series. If we read the sum in above formula as a sum modulo n, we can easily read the matching Renard-number just by multiplying the found number by 10.

So on the D-scale we read the Renard-number from the basic series (from 1 to 10):

$$\log(g_{n;k} \cdot g_{n;l}) = \log(g_{n;k}) + \log(g_{n;l}) = \frac{(k+l) \bmod n}{n}$$

and find the product by multiplying this value by 10. The same counts for the preferred numbers that are rounded off Renard-numbers.

Take for instance n = 20, k = 13 and l = 16. The corresponding preferred numbers are $g_{13} = 4.50$ and $g_{16} = 6.30$ respectively. Notice the matching mantissas are 13/20 and 16/20 respectively.

When we multiply the two preferred numbers, we find 28.35, which number has too many significant digits. Because the two numbers in the product have only 2 significant digits, the product of them cannot have more than 2 significant figures also. Therefore, we round the product off to 28.

We also have the simple equation:

$$(13 + 16) \bmod 20 = 9$$

from which we conclude we have to look for the 9th preferred number of the R20-series.
This numbers equals 2.8. In addition, by multiplying this value (that always lies in the interval from 1 to 10) by 10 we conclude the product must be 28.

We can notice the modulo-calculation of the sum of the position numbers k and l leads to the same slide rule movements, as we have to perform with multiplications on the C- and D-scale.

Because in divisions the mantissas of preferred numbers are subtracted, we find:

$$\log(\frac{g_{n;k}}{g_{n;l}}) = \log(g_{n;k}) - \log(g_{n;l}) = \frac{(k-l) \bmod n}{n}$$

Because the L-scale doesn't provide negative numbers, the value of the quotient

$$\frac{(k-l) \bmod n}{n}$$

has to be determined in such a way that the result is a non-negative integer number.

Thus, for all integers x we take:

$$0 \le x \bmod n < n$$

If, for instance, we divide the 9th by the 17th of the R40-series, then k – l equals –8. The point –8/40 isn't a value on the L-scale. However the point –8/40 corresponds with –8/40 + 40/40 = 32/40.
Hence

$$(9 - 17) \bmod 40 = 32.$$

The quotient of the 9th and the 17th number in de R40-series equals the 32nd number, of course divided by 10.

We conclude: what is above said about products of Renard-numbers applies mutatis mutandis to quotients.

Applications of NZ-rules

In this article we discussed some well known applications of which most come from the field of electronics. However, many other examples could be given. The interested reader should read the excellent articles about preferred numbers by Dr. Klaus Kühn and Eugen Paulin in Brief no.7 of the German RechenSchieber Treffen. See references [10] and [11].

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2. While writing this article, I found two excellent articles on the website of the German Circle of Slide Rule Collectors: www.IM2001.de. These articles, written by Dr. Klaus Kühn and Eugen Paulin, partly cover my article, but give also a great number of examples from physics and electro-technics that are not used in this article. See references [10] en [11]. Klaus Kühn gave me permission to copy the picture of the Aristo 89 NZ from his article.
3. In addition, I would like to thank my colleague Ing. Timmo Meyer, who is an inexhaustible source of knowledge about all kinds of preferred number applications in electronics and mechanics.

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